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RESEARCH ARTICLE

# Analysis of Instructional Support Functions of Virtual Manipulatives Using a Digital Typology Framework

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## ABSTRACT

The purpose of this study systematically analyzed the teaching and learning support features offered by virtual manipulatives, thereby providing fundamental data for the effective use of digital tools in mathematics education. Eleven highly utilized virtual manipulatives were selected. We analyzed their functionality according to four categories (lesson design, learner exploration, assessment and feedback, and instructor convenience), which reframe Choppin et al.'s (2014) digital typology to fit the context of virtual manipulative tools. The results revealed that most virtual manipulatives offered basic lesson design support features, such as curriculum integration, digital manipulative tool templates, and worksheets. However, features such as simultaneous collaborative learning, process recording, and teacher dashboards were only offered on some virtual manipulatives. The analytical framework and findings presented in this study provide practical guidance for teachers in selecting and utilizing virtual manipulative tools appropriate to their course objectives and learner characteristics.

**Key words :** Virtual manipulatives, digital typology, instructional support

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## Introduction

In mathematics education, manipulatives have long been recognized as key tools for supporting students' learning of abstract mathematical concepts through hands-on interaction with concrete objects. Historically, various physical manipulatives such as abaci, base-ten blocks, and geoboards have been used to promote students' conceptual understanding. These physical manipulatives have been found to enhance students' understanding and achievement, and to support long-term retention and transfer of mathematical content (D'Angelo & Iliev, 2012).

The proliferation of information technology and digital learning environments in the 21st century has extended the notion of manipulatives into virtual spaces. As interactive learning tools delivered via computers and the internet have been developed, mathematics teachers now draw on a wide variety of digital resources to design effective lessons. Virtual manipulatives, digital versions of physical manipulatives, are defined as “web-based visual representations of dynamic objects that present opportunities for constructing mathematical knowledge” (Moyer et al., 2002, p. 373). They provide manipulation experiences in digital environments that resemble those of physical manipulatives. Virtual manipulatives typically operate on the web or as dedicated applications, and users can directly manipulate on-screen objects (moving, transforming, etc.) via mouse or touch-screen interfaces.

With the rapid acceleration of digital transformation, a great number of virtual manipulatives have been developed and shared, and their types and functional scope have greatly expanded over the past three decades (Reiten, 2018). Early virtual manipulatives largely replicated the forms of physical manipulatives, but as computational power and dynamic visualization capabilities advanced, increasingly sophisticated virtual manipulatives were developed. These now provide representations and immediate feedback that are impossible with physical manipulatives (Durmu□ & Karakırık, 2006; Reiten, 2018). For example, the spectrum of virtual manipulatives ranges from familiar tools modeled on physical manipulatives (e.g., algebra tiles, fraction bars) to dynamic software for exploring function graphs and geometric constructions.

Virtual manipulatives tend to be visually engaging and game-like, intuitive to use, and accessible anytime and anywhere. Unlike physical manipulatives, they can be replicated without limit and used simultaneously by many learners (D’Angelo & Iliev, 2012). Owing to these characteristics, virtual manipulatives have been highlighted as powerful tools for designing effective mathematics learning environments in the digital era.

Virtual manipulatives are now used with increasing frequency in mathematics classrooms around the world, and numerous studies have reported their effectiveness for mathematics learning. For instance, virtual manipulatives can provide visual feedback on learners’ manipulation activities and offer hints that guide correct concept formation, thereby supporting self-directed learning even without direct teacher intervention (Voskoglou & Salem, 2020). Small-group or individualized instruction using virtual manipulatives has also been shown to significantly improve the mathematics achievement of low-performing students and students with special educational needs (Bouck et al., 2017; Park et al., 2022).

Durmuş and Karakırık (2006) examined whether virtual manipulatives could produce learning outcomes comparable to those of physical manipulatives and found that students using virtual manipulatives engaged deeply in tasks and achieved similar levels of performance. Virtual manipulatives can thus increase the variety of problems students can tackle, support the expression of abstract concepts, and promote equitable learning opportunities (Goldenberg, 2000; Reiten, 2018; Sarama & Clements, 2000).

Despite their potential to replace or complement physical manipulatives in classroom practice, compared to physical manipulatives, virtual manipulatives are more sensitive to technical issues and non-instructional factors. This makes the selection of appropriate tools and the development of instructional strategies tailored to specific educational contexts especially important. Although the importance of selecting and using virtual manipulatives has been discussed in prior research, existing studies have largely focused on (a) estimating learning effects or comparing virtual and physical manipulatives (e.g., Durmuş & Karakırık, 2006; Moyer-Packenham & Suh, 2012) or (b) surveying general trends and affordances of virtual manipulatives (e.g., Reiten, 2018; Voskoglou & Salem, 2020). Meanwhile, typological or evaluative frameworks for digital curriculum resources have been proposed (e.g., Choppin et al., 2014), and recent studies have adapted such frameworks to analyze digital mathematics resources in specific contexts (e.g., Son & Kang, 2024). However, there remains a lack of research that systematically compares multiple virtual manipulative virtual manipulatives through a common set of teacher- and student-facing instructional support functions—particularly those related to lesson design, task distribution and sharing, classroom orchestration, and formative assessment.

Addressing this gap, the present study applies a reconstructed digital typology framework to classify and compare the instructional support functions of widely used virtual manipulatives, thereby clarifying how different virtual manipulatives may support teachers' instructional decisions and students' mathematical exploration. We analyzed 11 virtual manipulatives using an analytical framework based on Choppin et al.'s (2014) digital typology. Digital typology provides a lens for analyzing the features of digital tools that may influence mathematics teaching and learning practices, and thus can be used to evaluate the pedagogical functions of virtual manipulatives and discuss directions for their use. The purpose of this study is not to directly infer the classroom effectiveness of virtual manipulatives. Rather, based on digital typology, it systematically analyzes the educational support functions of virtual manipulatives used in mathematics education. The purpose of this study is not to directly infer the classroom effectiveness of virtual manipulative tools. Rather, based on digital typology, it systematically analyzes the educational support functions of virtual manipulative tools used in mathematics education. We aim to provide a basis for teachers and teacher educators to select and implement virtual manipulatives that align with specific mathematical learning goals, and to derive implications for the effective instructional use of virtual manipulatives.

# Theoretical Background

## Virtual Manipulatives in Mathematics Education

Virtual manipulatives are interactive, screen-based mathematical objects intentionally designed to support manipulation (e.g., dragging, composing/decomposing, transforming, partitioning, iterating) and thereby enable learners to construct mathematical meaning through action. Following Moyer et al. (2002), we define virtual manipulatives as "web-based visual representations of dynamic objects that present opportunities for constructing mathematical knowledge" (p. 373). We therefore treat virtual manipulatives as a spectrum from physically-analogous tools (e.g., algebra tiles, geoboards) to born-digital manipulatives whose key affordances derive from computation (e.g., instantaneous linking of representations, replayable manipulation histories). This stance allows us to analyze instructional support functions based on what VMs enable in practice (representation, manipulation, sharing, and assessment), rather than on whether they mirror a physical tool. In this study, dynamic objects are specified as mathematically meaningful representations whose properties change in response to user actions while preserving explicit mathematical structure (e.g., maintaining equal partitions when resizing fraction bars). Therefore, not every interactive digital object qualifies as a virtual manipulative; the object must embed a mathematical representation and support manipulation that is interpretable in mathematical terms. For instance, a virtual geoboard can simulate the physical activity of stretching rubber bands over a pegboard to create shapes, while also providing additional features not possible with physical tools, such as extreme deformations of rubber bands or automatic shading of regions to highlight areas. In this way, virtual manipulatives enable extended forms of representation and manipulation that go beyond what is physically feasible.

We distinguish virtual manipulatives from broader digital learning resources (e.g., AI courseware, tutoring systems, general-purpose apps) in two ways: (a) virtual manipulatives foreground representation + manipulation as the primary learning mechanism, rather than sequence-driven instruction, content delivery, or gamified progression; and (b) virtual manipulatives are typically usable as standalone tools in multiple instructional designs rather than as fixed instructional modules. A virtual manipulative in this manuscript refers to a digital environment that hosts a library of multiple virtual manipulatives and provides shared access structures (e.g., categorization by grade/topic, search/browse menus), sometimes with additional teacher-facing supports (e.g., task sharing, classroom modes).

Key characteristics of virtual manipulatives in mathematics education include:

Dynamic interactivity: When a learner directly manipulates mathematical objects with a mouse or touch interface, the system immediately visualizes the result and provides feedback.

Multiple representations: The same concept can be represented in multiple forms (e.g., pictures, symbols, and language) and these representations can be linked to support understanding of abstract concepts.

Contextualization: Realistic scenarios or narratives can be embedded so that learners connect everyday experiences to mathematical ideas (Dick & Hollebrands, 2011; Zbiek et al., 2007).

Building on these characteristics, virtual manipulatives can function as cognitive tools that reduce learners' cognitive load and support engagement in higher-order thinking (Pea, 1987). For example, complex computations or geometric constructions can be handled by the software, allowing students to focus more on conceptual reasoning. Thus, virtual manipulatives can be used as cognitive supports that strengthen intuitive interaction and connect diverse representations to promote conceptual understanding.

## Educational Effects of Virtual Manipulatives

A review of research on the educational effects of virtual manipulatives suggests that their impacts can be grouped into three broad categories: cognitive outcomes, affective factors, and usefulness within teaching–learning environments.

### Cognitive Domains

In cognitive terms, virtual manipulatives play a central educational role in mathematics learning. Virtual manipulatives can directly engage learners' cognitive processes through visualization, manipulation, and feedback. These tools promote deep understanding and problem-solving by supporting high-level thinking.

Virtual manipulatives are particularly helpful when they connect complex mathematical concepts to concrete manipulation activities and enable learners to reconstruct concepts through multiple forms of representation. These features are highly useful for expanding learners' thinking and fostering connections among mathematical ideas.

Research has consistently shown that virtual manipulatives significantly enhance learners' spatial ability and mathematical reasoning. Son and Ryu (2016), for example, designed a lesson on "Understanding Three-Dimensional Shapes" for Grade 6 using a realistic mathematics education approach supported by three-dimensional virtual manipulatives. The results showed that students better understood complex spatial concepts such as rotation, nets, and cross-sections of solids. Virtual manipulatives helped overcome the limitations of two-dimensional textbook illustrations by offering opportunities for authentic spatial visualization. This experience in turn led to improvements in students' spatial visualization ability, spatial reasoning, and mathematical self-concept, as well as in their cognitive flexibility when formulating and solving mathematical problems.

Shin (2018) conducted a small-scale teaching experiment with students experiencing difficulty in mathematics to investigate the effects of the web-based Fun Fraction virtual manipulative on solving fraction multiplication word problems. Applying a Virtual–Representational–Abstract (VRA) instructional sequence, the study found significant gains in students' problem-solving accuracy, mathematical reasoning, and conceptual understanding. These findings indicate that virtual manipulatives can serve as cognitive scaffolds that allow learners to progress gradually from concrete manipulation to abstraction at a level appropriate to their understanding.

Moyer-Packenham and Suh (2012) compared the effects of virtual and physical manipulatives in Grade 5 fraction learning in the United States. Although achievement improved in all groups, the positive impact of virtual manipulatives was particularly pronounced among low-achieving students. The authors interpreted this as resulting from the visualization and immediate feedback functions that reduced cognitive load and supported concept formation. In another study, Bush (2021) examined the digital learning software Woot Math, which integrates virtual manipulatives, and reported that this environment improved students' understanding of fractions, operations, and problem-solving skills by providing level-appropriate feedback and tasks.

In sum, virtual manipulatives can help students understand mathematical concepts from multiple perspectives by linking different representational forms such as pictures, structures, and symbols (Lee & Yeo, 2024; Yeo & Webel, 2024).

**Affective Domains**

From an affective perspective, digital and virtual manipulatives can stimulate interest, confidence, and attitudes toward mathematics, thereby increasing persistence and engagement.

Park et al. (2015) examined grade 1 mathematics lessons using the game-based digital manipulative Playfacto. Their analysis showed that students increasingly perceived mathematics not as a difficult subject but as an enjoyable activity. Playful elements and repeated opportunities for manipulation fostered high levels of engagement and visibly enhanced students' confidence. These results highlight the positive role digital manipulatives can play in shaping affective competencies by providing repeated experiences of success in mathematics.

Reimer and Moyer (2005) reported that students found virtual manipulatives easy and enjoyable to use, and appreciated being able to try multiple strategies without fear of making mistakes. Students experimented with various mathematical outcomes in a low-risk context, and this process itself fostered intrinsic motivation. Synthesizing experimental studies published over a 15-year period, Moyer-Packenham et al. (2014) conducted a meta-analysis and found moderate positive effects of virtual manipulatives on affective outcomes such as interest, participation, and self-efficacy. Notably, participation went beyond simple behavioral engagement to encompass deeper involvement in problem situations, active thinking, and meaning-making.

Together, these studies show that digital environments can function not merely as technical media but as spaces for integrating students' emotions and attitudes into classroom activity. By creating learner-centered, interactive environments, virtual manipulatives can foster positive emotional dispositions toward mathematics and strengthen sustained motivation.

### Usefulness in Teaching–Learning Environments

With respect to their usefulness in teaching–learning environments, virtual manipulatives function as important mediators that can enhance the quality of mathematics instruction. The impact of virtual manipulatives is not confined to learners' manipulation activities; rather, their full potential is realized when they are integrated with teachers' instructional strategies. The introduction of virtual manipulatives can influence teachers' lesson-design practices, approaches to diagnosing student misconceptions and providing feedback, and the organization of collaborative learning structures, thereby promoting change across the teaching–learning process.

Song and Sung (2025) provided professional development for preservice elementary teachers focused on digital manipulatives and AI-based tools. Their findings showed statistically significant gains in participants' lesson-design skills and their ability to diagnose student errors, suggesting that digital tools can function as instruments for enhancing teachers' technological pedagogical content knowledge (TPACK)-based digital instructional competence, rather than merely as supplemental student tools.

Moyer-Packenham and Westenskow (2013) reported that virtual manipulatives not only provide individual learners with meaningful manipulation experiences but also foster collaborative work and mathematical discussion among peers. They argued that visual representations and the ability to store and share outcomes support mathematical communication, which in turn plays a critical role in eliciting students' mathematical thinking. These tools function effectively in cooperative learning contexts and enable differentiated instruction that accommodates diverse learners.

In short, the effectiveness of digital tools depends not only on their technical features but also on how they are integrated into teaching–learning activities. When supported by teachers' digital literacy and expertise in lesson design, virtual manipulatives can serve as catalysts for educational change that extend learning contexts and promote interaction.

## An Analytical Framework for Virtual Manipulatives

As digital resources for mathematics education have become widespread in classrooms, there is a growing need for frameworks that systematically analyze their structure and functions. To address this need, Choppin et al. (2014) proposed a digital typology for digital curriculum resources that classifies their features into three categories: program–student interaction, curriculum use and adaptability, and assessment system (Table 1). This framework is designed to evaluate whether digital resources transform the learning experience, provide teachers with flexibility in lesson design,

**Table 1.** Analytical framework for digital curriculum resources (Choppin et al., 2014)

| Category | Factor | Description |
|---|---|---|
| Program–student interaction | Learning experiences | Types of materials and activities students typically encounter in the program |
| | Differentiation/personalization | Features that allow selection or adaptation of content based on students' abilities |
| | Social/group features | Features that enable communication with teachers, parents, and other educational stakeholders |
| Curriculum use and adaptability | Reordering of curriculum | Functionality that allows teachers to map or sequence content in specific orders |
| | Lesson content design | Ability to select and assemble content for use in instruction |
| | Multimedia use | Availability of multimedia such as PowerPoint, videos, animations, and manipulatives |
| | Notes | Functions for storing and accessing notes taken during instruction |
| Assessment system | Assessment creation | Ability to create online assessments |
| | Storage of assessment data | Automatic analysis and storage of assessment results |
| | Dashboard | Provision of learning outcomes via dashboards |
| | Reporting | Creation and transmission of reports to parents and administrators |

and support rapid and automated assessment and reporting (Son & Kang, 2024).

The digital typology framework has been primarily used to analyze AI courseware. For example, Choppin et al. (2014) applied the framework to six AI courseware products widely used. They found that while some programs offered assessment systems that visually reported students' learning trajectories, many did not fully leverage the affordances of digital media. Similarly, Son and Kang (2024) analyzed 12 AI courseware products commonly used in Korea in terms of program–student interaction, curriculum use and adaptability, and assessment systems. Their study showed that although AI courseware offered distinctive features for teaching and learning, their functions for restructuring instruction were limited. These prior studies demonstrate that digital typology is an effective tool for evaluating digital programs from an instructional perspective.

Building on this work, the present study reconstructs the sub-factors of digital typology to suit the context of virtual manipulatives and proposes criteria for judging their instructional appropriateness and educational effectiveness. Using this reconstructed framework, we analyzed virtual manipulatives to provide teachers with a basis for selecting virtual manipulatives that align with their instructional goals and student characteristics.

# Methods

## Selection of Virtual Manipulatives

This study selected 11 virtual manipulative tools using the following three criteria: (1) tools designed around the curriculum of each country, (2) tools specifically for mathematics, excluding platforms that integrate game-based, curriculum-integrated, and learning functions, and (3) tools free of advertising or commercial elements. Using these criteria, we selected eleven virtual manipulatives (Table 2). The selected virtual manipulatives were STEAMuP Academy Mathland, Math Smart Manipulatives, MathCanvas, Polypad, Brainingcamp, Math Learning Center, Didax Virtual Manipulatives, National Library of Virtual Manipulatives, Mathsbot, Conceptua Math, and Toy Theater. To maintain a focus on manipulative-centered learning functions, game-based virtual manipulatives such as TokTok Mathematics Expedition, Math Playground, ABCya, and SplashLearn, as well as dynamic geometry programs such as GeoGebra, Desmos, and Cabri, were excluded from analysis.

**Table 2.** Virtual manipulatives for mathematics education

| Virtual Manipulatives | Types and Range of Tools | Target Grades and Domains | Supported Languages and Environment | Usage Policy |
|---|---|---|---|---|
| STEAMuP Academy Mathland | 41 digital tools (protractor, graphs, fraction bars, etc.) | Elementary–secondary / number & operations, fractions, geometry, algebra | Korean, English / web | Free |
| Math Smart Manipulatives | 80 digital tools (linking cubes, number cards, etc.) | Elementary / all strands | Korean / web | Free |
| MathCanvas | 46 digital tools (counters, pattern blocks, graphs, coordinate plane, etc.) | Elementary–secondary / all strands | Korean, English / web | Free |
| Polypad | 50+ digital tools (pattern blocks, algebra tiles, graphs, probability tools, etc.) | Elementary–secondary / geometry, number & operations, probability | Multiple languages / web | Free |
| Brainingcamp | 16 digital tools (base-ten blocks, fraction tiles and circles, algebra tiles, geoboards, etc.) | Elementary–secondary / number & operations, algebra, geometry | English / web, app | Paid |
| Math Learning Center | 10 digital tools (number lines, geoboards, base-ten blocks, etc.) | Early childhood–elementary / number & operations | English / web, app | Free |
| Didax Virtual Manipulatives | 19 digital tools (number lines, number models, dice, pattern blocks, etc.) | Elementary / number & operations, geometry | English / web | Free |
| National Library of Virtual Manipulatives | 100+ digital tools (number models, fraction models, graphs, geoboards, etc.) | Early childhood–secondary / number & operations, geometry, data | English / web | Free |
| Mathsbot | 31 digital tools (algebra tiles, base-ten blocks, geoboards, etc.) | Upper elementary–secondary (KS2–4) / all strands | English / web | Free |
| Conceptua Math | 10 digital tools (for natural numbers and fractions) | Lower elementary / fraction concepts | English / web, app | Free |
| Toy Theater | 150+ digital tools (geoboards, fraction bars, clocks, spinners, etc.) | Elementary / number & operations, geometry | English / web | Free |

## Analytical Framework and Procedure

To systematically compare the instructional support functions of the 11 virtual manipulatives, we reconstructed Choppin et al.'s (2014) digital curriculum resource framework for the context of virtual manipulatives. The central premise of digital typology is that instructional activity is shaped by feature-level affordances of digital resources (Choppin et al., 2014). While the original framework was developed for broader digital curriculum resources (including courseware), its categories can be meaningfully adapted to virtual manipulatives because virtual manipulatives use similarly involves (a) how learners interact with representations, (b) how teachers design and adapt tasks around those representations, and (c) how evidence of learning is recorded and made available for instructional decisions. However, because virtual manipulatives primarily focus on exploring mathematical concepts, they do not offer features like notes and reporting within the existing framework. Reflecting the instructional use of virtual manipulatives, we restructured the framework into three categories: (a) lesson design, (b) learner engagement and exploration, and (c) assessment and feedback and clarified the functional focus of each category (Table 3).

**Table 3.** Criteria for analyzing virtual manipulatives

| Category | Subcategory | Description |
|---|---|---|
| Lesson design | Curriculum alignment | Alignment of virtual manipulatives with national or regional curriculum standards and units to connect directly to lesson objectives |
| | Provision of activity materials | Provision of teacher resources such as guidelines, worksheets, sample tasks, and templates for using virtual manipulatives in lessons |
| | Task design and sharing | Features for creating activities with virtual manipulatives, assigning them as tasks, and sharing via links, codes, or LMS integration |
| Learner engagement and exploration | Multiple representations | Functions that enable exploration and transformation among different representations (e.g., equations, graphs, tables, geometric figures, tiles) |
| | Collaborative learning | Features supporting peer interaction and joint problem-solving (e.g., real-time co-editing, screen sharing, simultaneous manipulation and discussion) |
| Assessment and feedback | Process and product evaluation | Ability to store and replay manipulation processes and outputs so that teachers can assess students' work |
| | Student management | Operational features such as creating classes/groups, inviting and authenticating students, setting deadlines, tracking submissions, and reassigning tasks |

First, lesson design evaluates whether a virtual manipulative supports teachers in designing and implementing lessons through alignment with the curriculum. Specifically, we examined whether each tool is mapped to grade levels, mathematical content domains, or textbook units; whether the virtual manipulative provides activity materials (e.g., worksheets, lesson ideas, sample tasks, templates) that can be used in lesson preparation; and whether it offers functions for designing, editing, and assigning tasks to students.

Second, learner engagement and exploration assesses the extent to which virtual manipulatives support engaging and exploratory activities for learners. This includes whether students can go beyond following fixed procedures to design their own constructions and investigations; whether mathematical concepts or principles can be explored through multiple representations (e.g., equations, graphs, geometric figures, tables); whether collaborative learning is supported by sharing and co-editing manipulatives; and whether game-like elements (e.g., badges, points, missions) are provided to promote engagement.

Third, assessment and feedback examines how virtual manipulatives support evaluation and feedback on learners' manipulation processes and outcomes, as well as student management. We looked at whether virtual manipulatives record and replay manipulation histories and activity logs for formative and summative assessment, and whether they provide functions for class or student management and tracking learning histories.

In summary, the three categories of the analytical framework correspond to key aspects of instructional use—lesson design, exploration, and assessment—and form a mutually complementary structure. Two researchers independently examined each virtual manipulative's official documentation and used the virtual manipulatives directly to determine whether the features corresponding to each category were present. A feature was coded as present (√) only when it was executable in the user interface and its function was confirmed by documentation or on-screen descriptions; it was coded as absent/insufficient evidence (·) when it was not executable, did not meet the operational definition, or could not be corroborated. For instance, task design and sharing was coded as present when a teacher could create or duplicate an activity and distribute it via a link or QR code. Accordingly, MathCanvas was coded √ because teachers can author a canvas-based activity and share access through links/QR codes. Intercoder reliability was calculated using Cohen's kappa coefficient, and the resulting value exceeded .89 (Viera & Garrett, 2005), indicating high agreement. Disagreements were resolved through regular online meetings until full consensus was reached. Based on this process, we identified which criteria each virtual manipulative satisfied and selected representative virtual manipulatives within each category for illustrative case analysis.

## Results

Table 4 summarizes whether each of the 11 virtual manipulatives supports functions corresponding to the three categories and their subcategories. In this section, we compare and analyze virtual manipulatives characteristics by category and present representative cases.

**Table 4.** Instructional functions provided by virtual manipulatives

| Virtual Manipulatives | Lesson Design Support | | | Learner Engagement & Exploration | | Assessment & Feedback | |
|---|---|---|---|---|---|---|---|
| | Curriculum Integration | Activity Materials Provision | Task design and sharing | Multiple Representations | Collaborative Learning | Process and Product Evaluation | Student Management |
| STEAMuP Academy Mathland | √ | √ | √ | · | · | √ | · |
| Math Smart Manipulatives | √ | √ | · | · | · | · | · |
| MathCanvas | √ | · | √ | √ | √ | √ | √ |
| Polypad | · | √ | √ | √ | √ | √ | √ |
| Brainingcamp | · | √ | √ | √ | √ | √ | √ |
| Math Learning Center | · | √ | √ | √ | · | √ | · |
| Didax Virtual Manipulatives | · | √ | · | · | · | · | · |
| National Library of Virtual Manipulatives | √ | √ | · | · | · | · | · |
| Mathsbot | √ | · | · | · | · | · | · |
| Conceptua Math | · | · | · | √ | · | · | · |
| Toy Theater | √ | √ | · | · | · | · | · |

## Lesson Design Support

### Curriculum Integration

Six of the 11 virtual manipulatives explicitly align their virtual manipulatives with curricular structures. However, the specific ways in which manipulatives are linked to grade levels, topics, and content strands vary. STEAMuP Academy Mathland and Toy Theater guide users by grade level and show which manipulatives can be used at each level. MathCanvas and Mathsbot classify manipulatives according to mathematical topics. Math Smart Manipulatives and the National Library of Virtual Manipulatives categorize tools by both grade band and content strand.

For example, the National Library of Virtual Manipulatives organizes more than 100 manipulatives according to grade range and content strands such as number and operations, algebra, and geometry, thereby providing teachers with an overview of appropriate tools for specific curricular objectives (Fig. 1).


**Fig. 1.** Curriculum alignment of virtual manipulatives in the National Library of Virtual Manipulatives.

## Activity Materials Provision

Eight virtual manipulatives provide guidance or related content to help teachers understand how to use virtual manipulatives and develop lesson applications. These virtual manipulatives offer templates, worksheets, and sample activities linked to specific tools.

Polypad, for instance, provides example activities and templates associated with its manipulatives, allowing teachers to easily design learning activities by adapting existing materials. As shown in Fig. 2, teachers can leverage interface elements (e.g., a curated list of example assignments and templates linked to specific parishes) to start from pre-configured activities rather than having to create assignments from scratch.

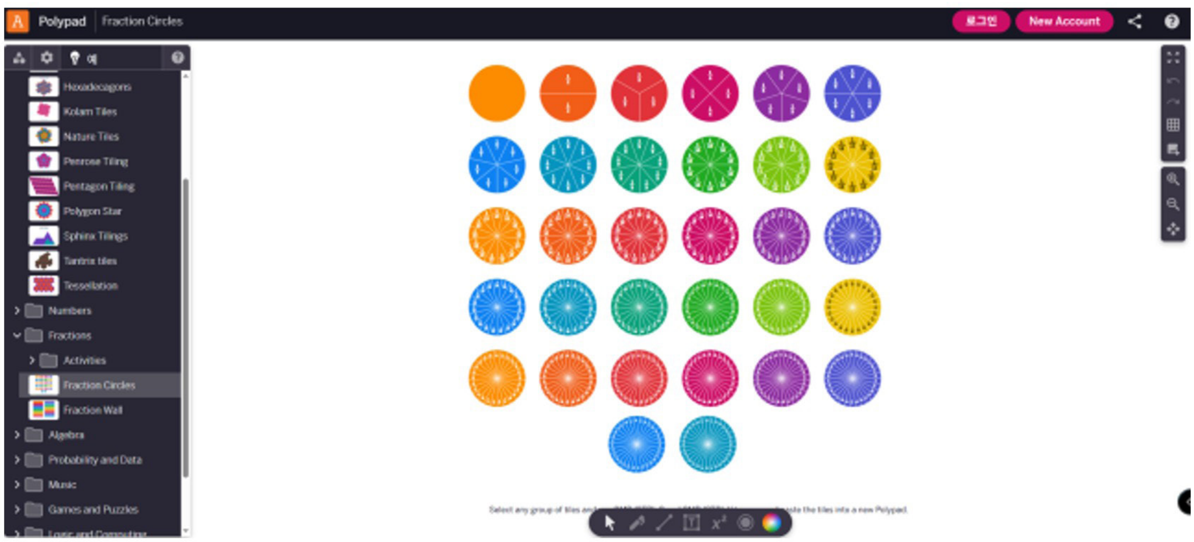


**Fig. 2.** Example of activity materials provided in Polypad.

STEAMuP Academy Mathland, Math Smart Manipulatives, Math Learning Center, Didax Virtual Manipulatives, National Library of Virtual Manipulatives, and Toy Theater all provide lesson ideas or downloadable worksheets for each manipulative. Math Smart Manipulatives and Math Learning Center allow teachers to download ready-to-use activity sheets, while Didax Virtual Manipulatives offers usage instructions and instructional ideas for each tool (Fig. 3).

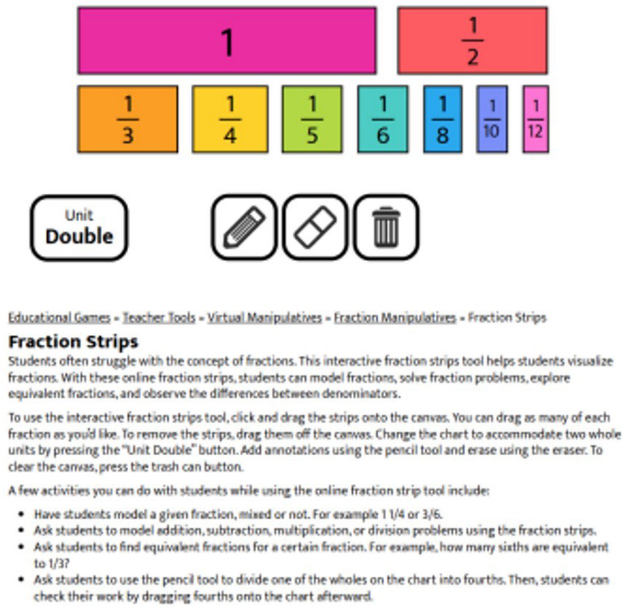


**Fig. 3.** Example of instructional ideas in Didax Virtual Manipulatives.

## Task Design and Sharing

Five virtual manipulatives (STEAMuP Academy Mathland, MathCanvas, Polypad, Brainingcamp, and Math Learning Centeroffer features for designing and sharing tasks based on virtual manipulatives.

MathCanvas, for example, allows teachers to create activities using virtual manipulatives and save them for classroom use (Fig. 4).

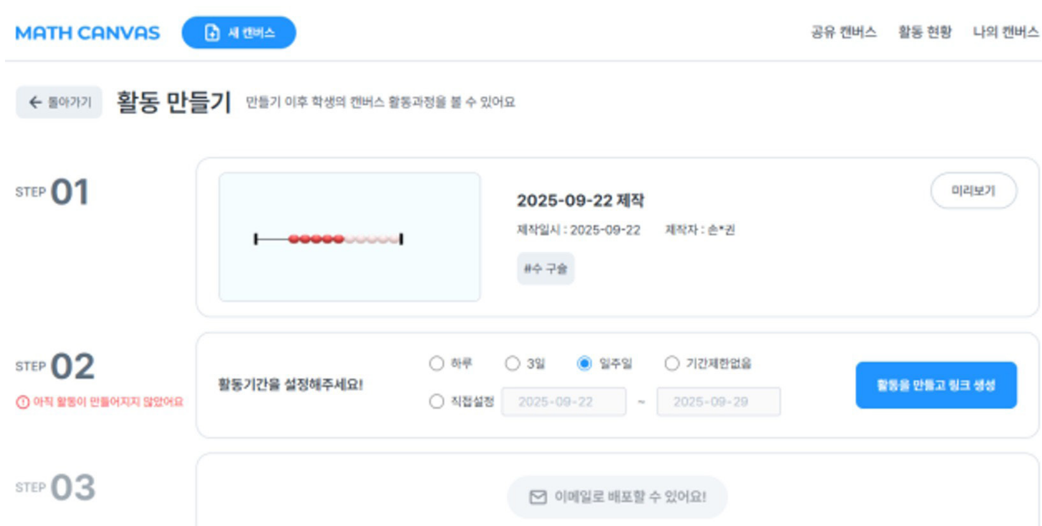


**Fig. 4.** Activity creation in MathCanvas.

Created activities can be shared with students via QR codes, links, or email. Math Learning Center, for instance, provides codes and links for sharing outcomes or tasks created with virtual manipulatives (Fig. 5).

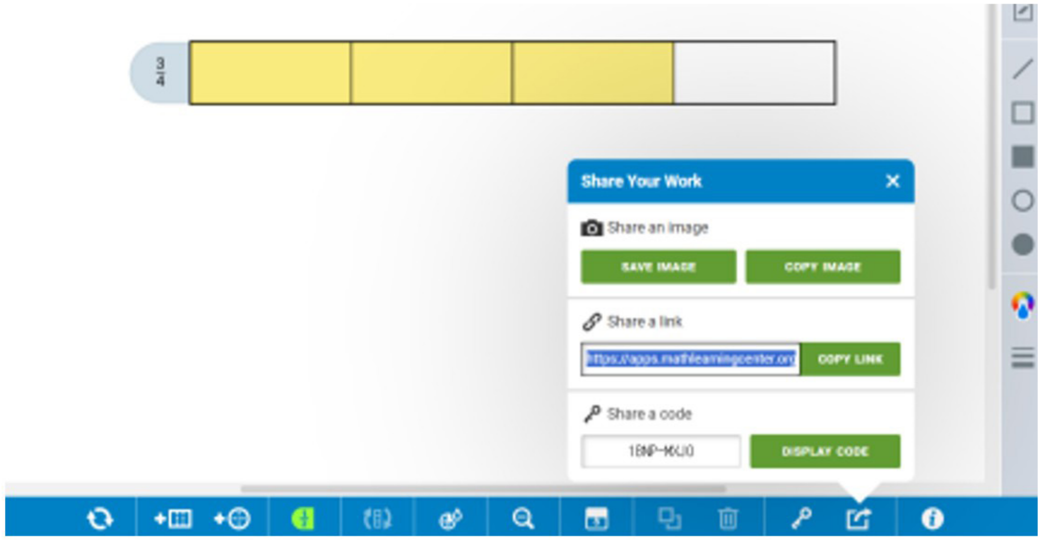


**Fig. 5.** Task sharing features in Math Learning Center.

## Learner Engagement and Exploration

### Multiple Representations

Five virtual manipulatives (MathCanvas, Polypad, Brainingcamp, Math Learning Center, and Conceptua Math) enable exploration of mathematical ideas through multiple, interconnected representations such as equations, graphs, tables, shapes, and prompts. These features allow teachers and students to visualize mathematical concepts and principles in various ways.

Polypad, for example, allows users to place algebra tiles on one part of the canvas, write corresponding algebraic expressions, and simultaneously display graphs of those expressions on a coordinate plane (Fig. 6).

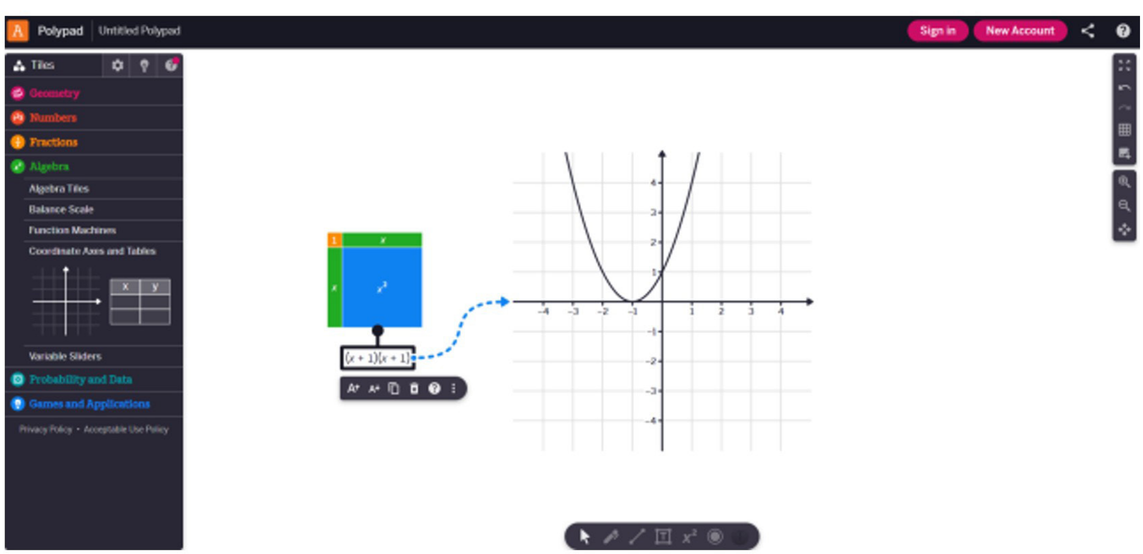


**Fig. 6.** Multiple representations in Polypad.

## Collaborative Learning

Real-time co-editing and screen-sharing features that support peer interaction are found in three virtual manipulatives: MathCanvas, Polypad, and Brainingcamp.

MathCanvas provides shared links through which users can view the same canvas and record or share each student's manipulation process. Polypad's Classroom mode enables teachers and students to work simultaneously on a shared virtual canvas, thus supporting real-time collaboration between students and between teacher and students. Brainingcamp offers a LIVE Session feature that allows teachers to observe students' work in real time, provide feedback, and even enable simultaneous manipulation (Fig. 7).

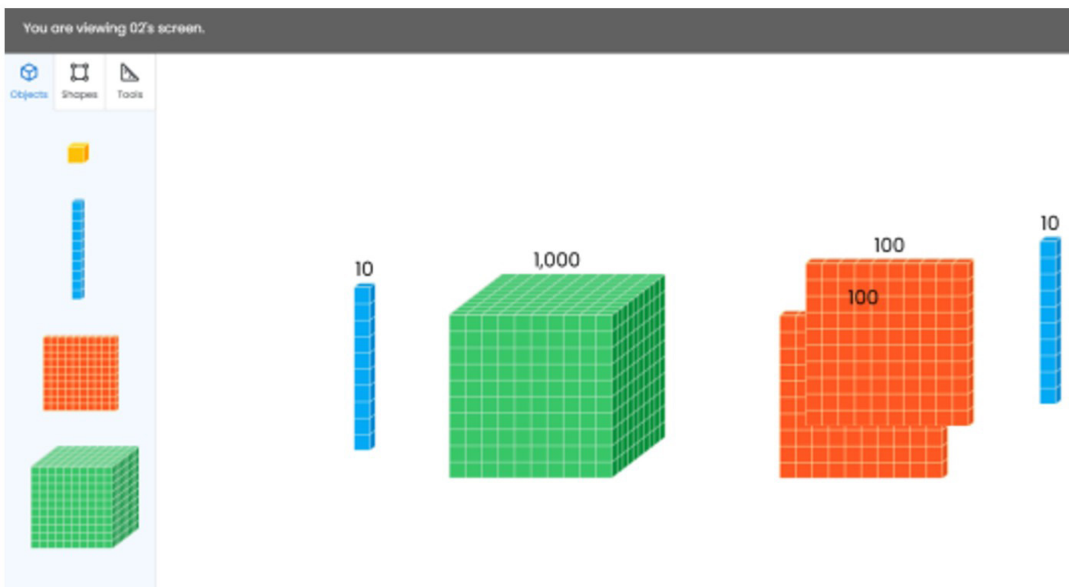


**Fig. 7.** Collaborative learning through Brainingcamp's LIVE Session feature.

## Assessment and Feedback

## Process and Product Evaluation

Five virtual manipulatives (STEAMuP Academy Mathland, MathCanvas, Polypad, Brainingcamp, and Math Learning Center) support evaluation of students' work using recordings or captures of manipulative activities.

STEAMuP Academy Mathland, Polypad, and Math Learning Center enable students to save final products as image files and, when needed, compare intermediate versions to trace changes in strategies and solution paths. In contrast, MathCanvas and Brainingcamp provide functionality for observing activities in real time or replaying activity histories.

In LIVE mode, Brainingcamp allows teachers to observe individual manipulation processes in real time and conduct process-oriented assessment. MathCanvas provides a replay function for manipulation sequences, allowing teachers to review each student's activity and perform process-based evaluation (Fig. 8).

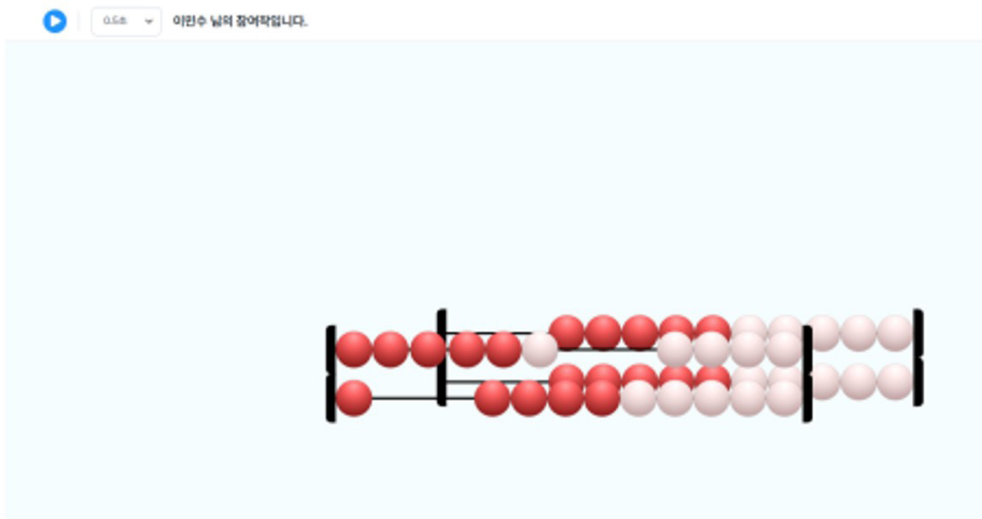

**Fig. 8.** Replay of manipulation processes in MathCanvas.

## Student Management

Three virtual manipulatives (MathCanvas, Polypad, and Brainingcamp) support student management functions that allow teachers to monitor students' use of virtual manipulatives.

MathCanvas enables teachers to monitor students' use of manipulatives based on submitted activities. Polypad's Classroom mode allows teachers to distribute and collect tasks for individual students. Brainingcamp provides a dashboard within the LIVE Session mode that displays each student's task progress at a glance and allows teachers to send messages or pause students' work (Fig. 9).

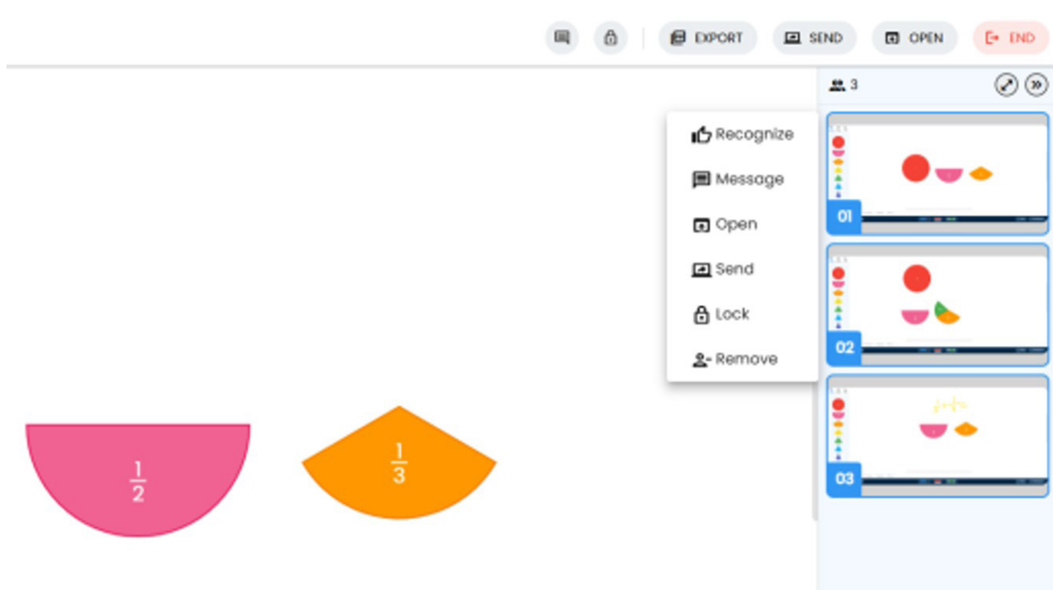


**Fig. 9.** Student management dashboard in Brainingcamp.

### Result Summary

Across virtual manipulatives, three overarching patterns were observed. First, curriculum integration tended to be emphasized in virtual manipulatives positioned for school use, whereas tool libraries without explicit curriculum mapping often prioritized breadth of manipulatives and ease of access. Second, virtual manipulatives that supported task design and sharing were also more likely to provide collaboration or classroom orchestration features, suggesting a shift from 'stand-alone manipulatives' toward teacher-mediated activity systems. Third, assessment-related functions were unevenly distributed: while several virtual manipulatives allowed saving or exporting student artifacts, only a small subset provided process evidence (e.g., activity histories) or teacher-facing monitoring that can directly support formative assessment decisions.

## Discussion and Conclusion

This study reconstructed Choppin et al.'s (2014) digital typology into three categories—lesson design support, learner engagement and exploration, and assessment and feedback—and systematically analyzed 11 virtual manipulatives. The main implications of the findings are as follows.

First, by applying Choppin et al.'s framework to the context of virtual manipulative tools, this study presents evaluation criteria for systematically assessing their instructional support capabilities for mathematics education. Whereas the original framework included several sub-factors tailored to courseware (e.g., multimedia use, reordering of learning paths), the reconstructed framework more explicitly captures the core features of virtual manipulatives: lesson design, learner engagement and exploration, and assessment and feedback. Rather than claiming a validated theory of instructional effectiveness, our contribution is to provide transparent, replicable criteria for identifying and comparing support functions that matter for teaching mathematics at the level of individual virtual manipulatives and their provision. This reframing positions the framework as a systematic descriptive analytic instrument for comparative analysis, and it clarifies the scope of inference that can be warranted from feature inspection alone.

Therefore, this analytical framework can provide teachers with practical criteria for selecting virtual teaching aids and evaluating whether each virtual teaching aid has the capabilities to support lesson design. It not only foregrounds the functional characteristics of digital tools but also illuminates their pedagogical potential (Song & Sung, 2025). For example, teachers can use the framework to distinguish between virtual manipulatives that merely display fractions and those that support rich exploratory tasks, multiple manipulation pathways, collaborative learning, and in-depth investigation of fraction concepts.

Second, the analysis showed that virtual manipulatives offer distinctive functions across the three categories of lesson design support, learner engagement and exploration, and assessment and feedback. For instance, STEAMuP Academy Mathland, Math Smart Manipulatives, MathCanvas, Math Learning Center, Didax Virtual Manipulatives, the National Library of Virtual Manipulatives, and Toy Theater all provide guidance linking manipulatives to grade levels, topics, and units. Most virtual manipulatives also provide sample worksheets and templates that teachers can adapt for lessons, thereby supporting the instructional use of virtual manipulatives aligned with lesson objectives and activities.

In contrast, only a subset of virtual manipulatives offer task-design and sharing features, multiple representations, collaborative learning functions, and assessment and student-management tools. Virtual manipulatives such as MathCanvas, Polypad, and Brainingcamp stand out in this regard, enabling teachers to design and share tasks, support real-time co-editing and screen sharing for collaborative inquiry, and record and replay manipulation processes through teacher dashboards.

Taken together, these findings indicate that while most virtual manipulatives support curriculum alignment and provide activity guidance, relatively few fully exploit the potential of digital media in terms of task design, learner engagement, and formative assessment. This aligns with prior research that has argued many digital programs do not take full advantage of the unique affordances of digital media (Choppin et al., 2014; Son & Kang, 2024).

To use virtual manipulatives effectively in schools, teachers must clearly understand the functional characteristics of each virtual manipulative and select those that match instructional goals and learner needs, while strategically combining the strengths of multiple virtual manipulatives when necessary. At the same time, there is a need for further functional expansion to enhance the educational utility of virtual manipulatives. Future virtual manipulatives development should consider expanding features such as teacher-controlled task design, collaborative learning tools, learning analytics and adaptive feedback, and report generation linked to teacher rubrics, thereby extending the educational potential of virtual manipulative environments.

This study has several limitations. First, we focused on analyzing the functional characteristics of virtual manipulatives and did not empirically examine how these functions are used in actual classrooms or what learning outcomes they produce. Functional sophistication does not guarantee instructional effectiveness, as outcomes are mediated by teachers' instructional strategies, students' prior knowledge, and classroom context. Future research should therefore build on the present analytical framework to implement selected virtual manipulatives in real classrooms and empirically examine their effects on students' achievement, attitudes toward mathematics, and problem-solving abilities.

Second, our analysis was limited to eleven virtual manipulatives, whereas in practice many more virtual manipulatives are available. In particular, virtual manipulatives that integrate adaptive, AI-based learning systems have recently emerged. Future research should expand the range of virtual manipulatives analyzed, including systems with advanced AI-driven personalization features, to provide a more comprehensive picture of the landscape of virtual manipulatives.

Despite these limitations, this study provides practical criteria for evaluating the instructional support functions of virtual manipulatives and offers a comprehensive analysis of 11 virtual manipulatives. The findings deepen our understanding of digital tools and suggest ways to use virtual manipulatives effectively in mathematics education. The results confirm that each virtual manipulative provides a unique configuration of functions, and that teachers and teacher educators need to assess virtual manipulatives using the proposed framework and select or combine them according to instructional goals and learner characteristics.

Maximizing the educational impact of virtual manipulatives requires attention not only to their technical features but also to curriculum alignment, support for lesson design, real-time feedback on learning processes, and the creation of collaborative inquiry environments (Choppin et al., 2014). The results of this study demonstrate the capabilities of virtual teaching aids for mathematics instruction. Manipulative tools with curriculum alignment and ready-made activity materials can lower teachers' barriers to entry and facilitate effective lesson preparation. Manipulative tools with the coordination, structure, and flexibility of multiple representational methods can provide an environment for students to explore mathematics. This may require greater coordination and clear assignments from teachers. Emerging AI-based tools are increasingly incorporating features such as automated interpretation of activity traces, adaptive hints, and analytical dashboards to teach mathematics. Therefore, future research should examine the differences between AI-based tools and existing digital tools, how teachers' educational decisions interact with the tools, and how the tools' features transform their teaching methods. Given the proliferation of virtual manipulatives in today's educational landscape, effective classroom integration will depend on accurately diagnosing each virtual manipulative's functional affordances and limitations and systematically orchestrating them in ways that are attuned to specific instructional contexts. Continued practical and theoretical discussions are needed to support such orchestration.

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