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RESEARCH ARTICLE

# Analysis of Students' Eye Movement and Understanding Based on the Presentation Methods of Visual Models for Addition and Subtraction Algorithms

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## ABSTRACT

Addition and subtraction of natural numbers, including carrying and borrowing, are concepts that are difficult for students to understand; therefore research on the presentation method of visual models for effective learning is necessary. This study classified the presentation methods of visual models into two types through a review of the literature and analyzed students' eye movements and understanding according to the differences in presentation methods, in order to explore which visual model presentations are most effective for students' attention and understanding. The results of the study confirmed that students' gaze patterns and understanding differed depending on the visual model presentation method, and revealed that a specific presentation method increased students' attention focus and comprehension. Based on these results, this study suggests educational implications for improving the design of visual models in textbooks and for enhancing the teaching and learning of addition and subtraction of natural numbers.

**Keywords:** Visual model, eye-tracking, presentation methods, addition and substraction

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## Introduction

The comprehension of abstract mathematical concepts presents a formidable challenge for elementary students, with visual models serving as critical pedagogical tools in mathematics textbooks to facilitate the concretization of these concepts (Reys et al., 2015; Van Garderen et al., 2018). Nevertheless, prior research has identified potential limitations associated with visual models: excessive complexity or inadequate explanation may precipitate misconceptions or impede students' grasp of mathematical principles (Yu & Chang, 2021; Lee, 2022). The operations of addition and subtraction with natural numbers, particularly those necessitating carrying and borrowing, are recognized as particularly arduous for young learners. To mitigate these difficulties, textbooks frequently integrate visual models, such as base-ten block manipulations, alongside numerical expressions to elucidate the conceptual underpinnings and procedural mechanics of addition and subtraction algorithms. Despite these efforts, empirical evidence suggests that students often encounter challenges in interpreting these visual models and establishing meaningful connections with the corresponding algorithmic processes (Lim & Chang, 2019).

Given these pedagogical challenges, there is an emergent imperative to rigorously evaluate the efficacy of visual models in enhancing students' conceptual understanding and to delineate presentation methods that optimize clarity and coherence (Kim & Choi, 2019). To address this issue, this study utilized eye-tracking technology as a primary analytical tool, which quantitatively and objectively measures learners' attentional distribution and cognitive processing (Holmqvist & Andersson, 2017; Rayner, 1998). Previous studies using eye-tracking technology in mathematics education have investigated learners' cognitive and affective processes, attention, or intentions without interfering with learners' problem-solving processes or intervening in mathematics instruction (Schindler et al., 2025), such as fraction problem solving (Obersteiner & Tumpek, 2016), proof-reading strategies (Inglis & Alcock, 2012), and novices' and experts' perceptual processes (Inglis & Alcock, 2012; Krichevets et al., 2014). However, few studies have examined learners' cognitive processing of visual models in mathematics textbooks using eye tracking.

This study systematically classified the presentation methods of visual models used in official mathematics textbooks and analyzed students' gaze patterns and levels of understanding according to various visual presentation methods. We aimed to suggest evidence-based strategies for effective design and improvement of visual models through empirical analysis. In particular, we focus on analyzing the gaze differences in the decimal block visual models of natural number addition and subtraction algorithms. These research results can contribute to improving the teaching and learning of natural number addition and subtraction and to revealing how students connect visual models with algorithms, as well as providing initial information for the development of future mathematics textbooks.

# Theoretical Background

## Visual Models Explaining Addition and Subtraction Algorithms

The standard algorithm for addition and subtraction refers to a systematic procedure that begins with the ones place on the right and proceeds leftward, calculating each place value performing carrying or borrowing when the result of a place value exceeds or falls below ten (Chang, 2014). In South Korean mathematics textbooks, visual models involving the manipulation of base-ten blocks are meticulously integrated with computational expressions to facilitate learning of the standard algorithms for addition and subtraction. An analysis of the explanation of the three-digit addition algorithm with carrying, as delineated in the 2015 revised curriculum certified textbooks, reveals a structured approach, as illustrated in Figure 1. The majority of textbooks elucidate the addition algorithm by organizing visual models of base-ten blocks and computational expressions into three distinct stages. However, Textbook ⑥ presents the manipulation of base-ten blocks in a single illustration without dividing it into three stages, introducing the algorithm through computational expressions. Additionally, Textbooks ①, ④, and ⑤ highlight the visual models with background colors corresponding to the algorithm's stages, whereas Textbooks ② and ③ lack such highlights. With respect to the orientation of the algorithm's presentation, most textbooks, including Textbooks ①, ②, ③, and ⑤, adopt a vertical arrangement of the visual model and computational expression, with the calculation process depicted horizontally from ones to tens to hundreds. In contrast, Textbook ④ positions the visual model and computational expression in a side-by-side configuration, presenting the calculation process vertically from ones to tens to hundreds.

Based on a comparative evaluation of the certified textbooks, the variations in the presentation methods of visual models were systematically classified into two primary dimensions: the presence or absence of visual highlighting and the directionality of presentation.

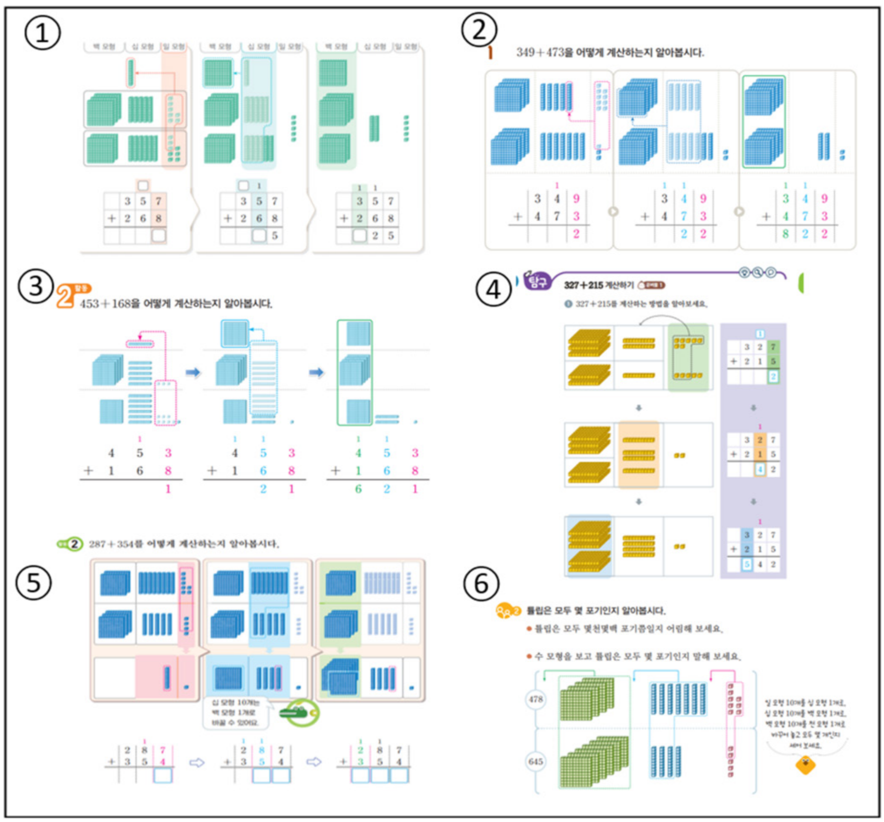



**Fig. 1.** Explaining 3 digit addition algorithm with carrying in the certified textbooks

## The Multimedia Instructional Design Principles

Mayer (2021) proposed 15 principles for designing educational materials to optimize cognitive load in the learning process through multimedia, based on the Cognitive Theory of Multimedia Learning (CTML), which considers learners' limited cognitive capacity (Mayer, 2021). Among these, the principles relevant to this study are the signaling principle and the spatial contiguity principle. The signaling principle posits that students learn more effectively when cues are added to highlight the structure of essential content in instructional materials. In this study, we analyze the differences between visual models where the manipulation process of base-ten blocks and corresponding parts of computational expressions are highlighted with background colors and those without such highlights. The spatial contiguity principle states that learning is enhanced when related text and images in instructional materials are presented in close proximity rather than far apart. In this study, when visual models are presented horizontally, the key areas of the visual model and computational expression are positioned closely together, whereas when presented vertically, these key areas are relatively farther apart.

### Eye Tracking Analysis

Eye-tracking methodology constitutes a sophisticated research paradigm that systematically monitors the eye movements of participants to elucidate patterns of visual attention, thereby facilitating profound insights into cognitive processing (Bojko, 2013; Holmqvist & Andersson, 2017). The non-invasive nature of eye-tracking, achieved through the precise measurement of corneal reflections, ensures participant safety while enabling the continuous assessment of dynamic cognitive states (Holmqvist & Andersson, 2017). Consequently, this methodology has been extensively adopted across diverse disciplines, including cognitive psychology, media studies, design, and educational research, to investigate the intricacies of cognitive engagement.

The theoretical underpinning of eye-tracking research is encapsulated in the eye-mind hypothesis, which postulates that an individual's gaze location is a direct manifestation of their cognitive focus and attentional allocation (Just & Carpenter, 1976). Eye-tracking data are primarily delineated into two categories: fixations, characterized by sustained gaze on a specific point, and saccades, denoting rapid eye movements between points. Fixations are indicative of heightened significance, relevance, or interest in the gazed-upon stimulus. The duration of fixations serves as a proxy for cognitive load and effort, with protracted fixation durations suggesting elevated cognitive investment and abbreviated durations reflecting fleeting attentional engagement (Bojko, 2013). Saccadic movements, conversely, signify transitions in attentional focus (Van Gog & Jarodzka, 2013).

Notably, gaze patterns may not invariably mirror cognitive processes, particularly during introspective activities such as memory recall or strategic planning. To mitigate this limitation, researchers often employ triangulation by integrating eye-tracking with qualitative methodologies, such as structured interviews or think-aloud protocols, to enhance interpretive validity (Schindler & Lilienthal, 2019).

For analytical purposes, researchers delineate Areas of Interest (AOIs) congruent with the study's objectives, enabling quantitative evaluation of fixation duration and frequency within these regions. Visualization tools such as Heatmaps and scan paths can illustrate the distribution, frequency, and duration of fixations (Shin & Shin, 2016).

## Methods

### Participants

The participant cohort comprised 41 third-grade students from two classes at Y Elementary School in Sejong City, all of whom provided informed consent for participation in the experiment. One class was assigned to investigate the presence or absence of highlighting in visual models, while the other class examined the directionality of visual model presentation, with distinct assessment tools administered accordingly. Given that the assessment tools were developed in two types, each corresponding to two variations of visual model presentation methods, the students in each class were divided into two homogeneous groups. This division was informed by individual characteristics, including unit assessment scores for three-digit addition and subtraction, gender, and consultations with the class teachers. The mean unit assessment scores for each group are presented in Table 1.

**Table 1.** Average of group evaluation scores

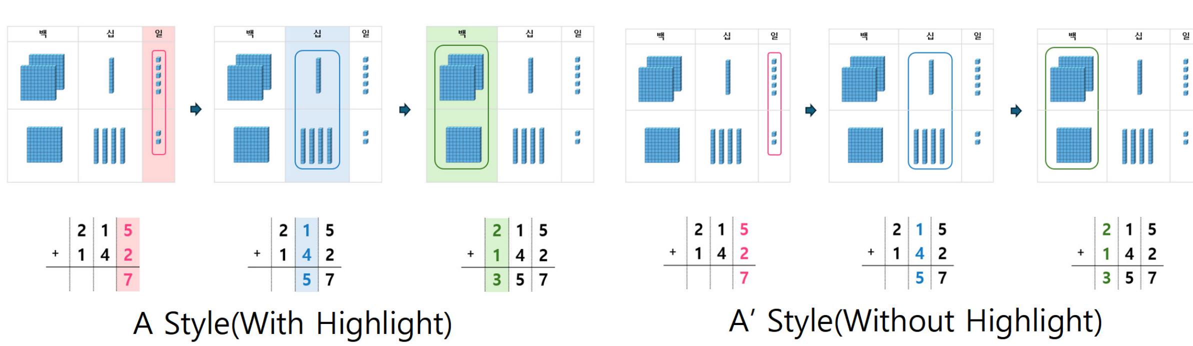
| Research question | Group | N | M | SD | t | p |
|---|---|---|---|---|---|---|
| Presence or Absence of Highlighting in Visual Models | A (With) | 11 | 8.23 | 1.29 | -.196 | .847 |
| | A' (Without) | 10 | 8.35 | 1.58 | | |
| Direction of Visual Model Presentation | B (Horizontal) | 10 | 8.65 | 1.45 | -.171 | .866 |
| | B' (Vertical) | 10 | 8.75 | 1.14 | | |

Among the students participating in this study, one student exhibited an eye-tracking rate of 80% or lower and was consequently excluded from the eye-tracking data analysis. Additionally, in instances where students demonstrated high eye-tracking rates but failed to generate eye-tracking data for specific tasks, such cases were excluded from the analysis solely for the affected tasks.

## Procedure

### Task

To investigate the differential effects of the presence or absence of the highlight and direction of visual model presentation, this study developed two distinct types of assessment tools, each tailored to a specific presentation method for each condition. The assessment tools comprised a total of six tasks: three addition tasks and three subtraction tasks. These tasks were structured to align with the lesson sequence of the three-digit addition and subtraction unit in the curriculum, encompassing the following: three-digit addition without carrying (e.g., 215 + 142 = 357), three-digit addition with one instance of carrying (e.g., 237 + 244 = 481), three-digit addition with two instances of carrying (e.g., 148 + 365 = 513), three-digit subtraction without borrowing (e.g., 349 - 214 = 135), three-digit subtraction with one instance of borrowing (e.g., 326 - 154 = 172), and three-digit subtraction with two instances of borrowing (e.g., 423 - 156 = 267). The validity of the assessment tool was verified by two mathematics education experts and two in-service teachers with extensive experience in mathematics task development. The assessment tools were created using Microsoft PowerPoint and presented to students on a 24-inch monitor screen. An example of the assessment tool is illustrated in Figure 2.



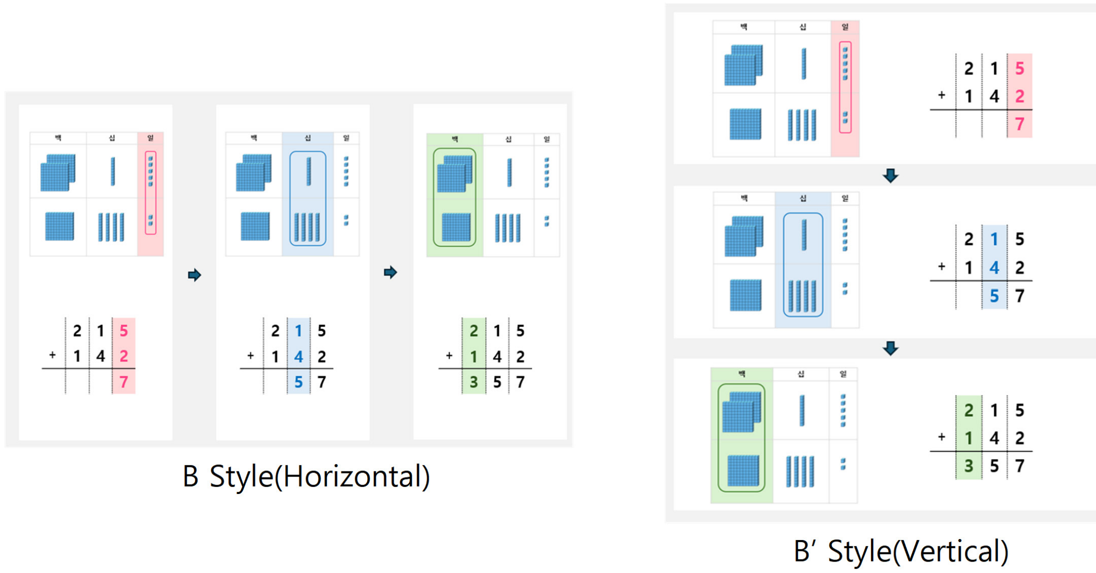



**Fig. 2.** Example of a test tool

### Experiment

The experiment in this study consisted of the eye tracking test and understanding assessment corresponding to each research question. The eye tracking tests analyzed students' visual attention and cognitive processes by examining which areas of visual models attracted their gaze, how long their gaze remained fixed on these areas, and the gaze trajectories between regions. Following the eye tracking test, students' understanding was assessed through the base-ten block test and interviews. The experimental procedure is detailed in Table 2.

**Table 2.** Experimental procedure

| Eye tracking test | | Manipulation test of number models | | Interview |
|---|---|---|---|---|
| Collect student gaze data by having students wear an eye tracker and look at the instrument presented on the monitor screen | → | Present a task to perform manipulations like the visual model in the eye tracking test by presenting a calculation formula | → | Check understanding of the components of the visual model and the principles of each step of the process |

The eye tracker used in this study was the Tobii Glasses 3, a wearable eye tracking device. Students wore the eye tracker while visual tasks were presented on a monitor, and gaze data was collected as they explored the tasks. The assessment tool consisted of six tasks. Students were instructed to observe each task carefully for at least 10 seconds and say "Next" once they felt they had achieved understanding. Upon the student's verbal cue, the researcher clicked the mouse to present the next task. Since there was no time limit, the duration of task completion varied among participants.

Understanding was assessed through base-ten block tasks and interviews. The base-ten block tasks included one addition problem with carrying and one subtraction problem with borrowing, both of which had been presented during the eye tracking test. Students were asked to perform the calculations using base-ten blocks, replicating the visual models they had previously seen. The interviews followed the base-ten block tasks and involved presenting paper-based visual scenes of an addition problem involving two instances of carrying (148 + 365 = 513) and a subtraction problem involving two instances of borrowing (423 - 156 = 267).

In the interview protocol, the researcher asked participants to describe their understanding of what they observed in the eye-tracking test. Participants observed their own gaze records through retrospective reflection and then talked about their thoughts about the visual model. Participants' gaze records serve as memory cues that allow participants to express their thoughts in previous activities (van Gog et al., 2009). If a student had difficulty articulating their thoughts, the researcher guided them through each step of the visual model, providing prompts to elicit the meaning of each stage.

**Fig. 3.** Scene during research execution

### Data Analysis

Students' gaze data were analyzed using Tobii Pro Lab, the software provided by the manufacturer of the eye tracker, Tobii, along with the statistical software SPSS 29.0. In Test types A and A', areas with and without visual emphasis in the visual models and numerical expressions were each designated as Areas of Interest (AOIs). Each task in both forms contained six AOIs of equal size.

For Test types B and B', the AOIs were defined as rectangular regions covering each step's visual model and corresponding numerical expression. This setup aimed to analyze the degree of gaze fixation and the patterns of gaze shifts depending on the presentation direction of the visual models. The defined AOI regions and their labels are shown in Fig. 4.

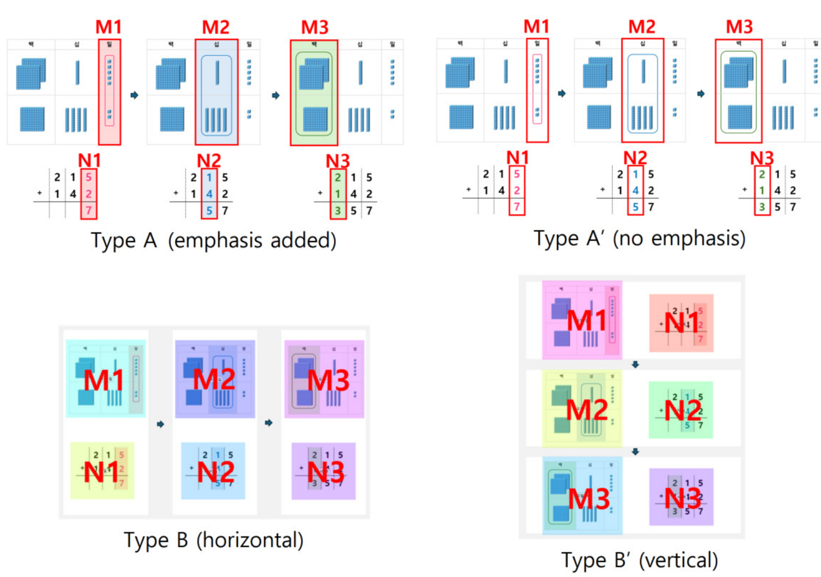


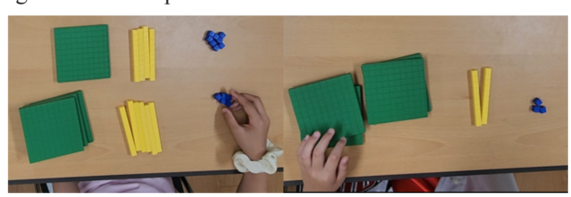
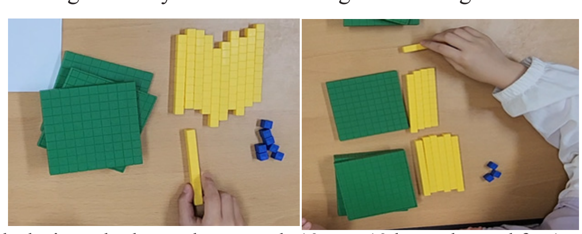
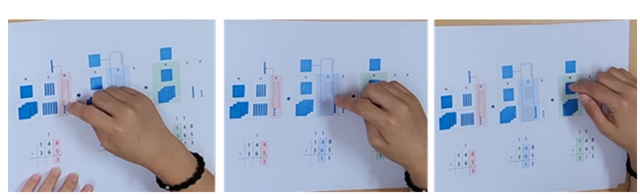

**Fig. 4.** AOI setup example

In this study, students' understanding was assessed based on three evaluation criteria: manipulation of base-ten blocks, understanding of carrying and borrowing, and understanding of the standard algorithm procedure. Students were considered to have adequately understood the visual models representing the standard algorithms for addition and subtraction only if they met all three criteria.

The first two criteria—manipulation of base-ten blocks and understanding of carrying and borrowing—were evaluated based on both the base-ten block tasks and the interview responses. For the third criterion—understanding of the standard algorithm procedure—students were assessed during the interviews on whether they could accurately explain each step of the visual model. This judgment was made with consideration of prior findings suggesting that manipulating the largest place value first may feel more intuitive for students, even if it does not align with the standard algorithm (Im & Jang, 2019).

The analysis was conducted by the primary researcher. In cases where the judgment was unclear, evaluations were discussed and determined through a seminar involving one expert in elementary mathematics education, two doctoral students, and three master's students. The three evaluation criteria and examples are presented in Table 3.

**Table 3.** Evaluation factors of understanding test

| Assessment elements | Contents and examples | |
|---|---|---|
| Model manip ulation | Base-ten blocks manipulation task | Does the student accurately represent numbers using a base-ten blocks and perform operations such as adding or removing models of each place value in addition and subtraction?  |
| | Interview | In the interview, can students read numbers when they see a visual representation of base-ten blocks and interpret the meaning when they see scenes of adding or subtracting? |
| Carry and borrow | Base-ten blocks manipulation task |  If the base-ten blocks in each place value exceeds 10, can 10 be exchanged for 1 model of the upper digit (carrying), or if subtraction is not possible, can 1 model of the upper place value be exchanged for 10 models of the corresponding place value (carrying)? |
| | Interview | In an interview, can the student explain the carrying and borrowing represented in the visual model? |
| Understanding standard algorithm procedures | Interview | Can you explain step-by-step by looking at the visual model that the first step represents the operation of the one-model, the second step represents the operation of the ten-model, and the third step represents the operation of the hundred-model?  |

# Results

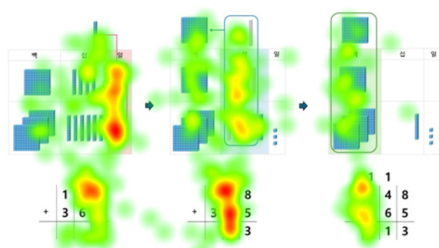
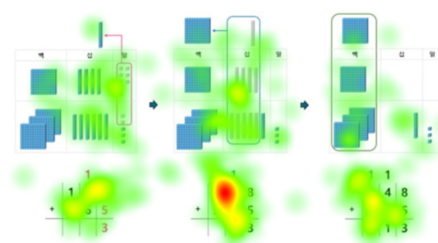
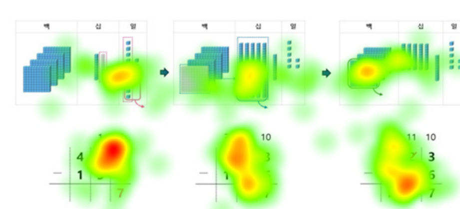
## Presence or Absence of Emphasis in Visual Models

### Eye-tracking Data Analysis

The results of the heatmap comparison between the two groups are presented in Table 4. Group A showed strong gaze fixation on the areas of the visual model that were highlighted with pink, light blue, and light green to emphasize each step of the algorithm. In contrast, Group A’, which was presented with visual models without such emphasis, exhibited a more evenly distributed pattern of gaze fixation across the entire visual model.

When comparing the heatmaps of the numerical expressions presented at the bottom of the tasks, it was found that Group A’ generally showed stronger gaze fixation on the equations than Group A. Furthermore, while Group A demonstrated efficient gaze fixation centered around the emphasized steps of the algorithm within the equations, Group A’ showed a more dispersed gaze pattern across the entire equation, likely due to the absence of step-specific visual emphasis.

**Table 4.** Heatmap of group A and group A'

| | A (emphasis added) | A’ (no emphasis) |
|---|---|---|
| TAddition3 |  |  |
| Subtraction3 |  |  |

The average fixation count (FC) on the predefined AOIs was examined for both groups. Table 5 presents the group-wise comparison of the total mean fixation counts across all six tasks. Group A, which was provided with visual emphasis, showed a higher average fixation count ($M = 116.10$) compared to Group A’, which received visuals without emphasis ($M = 79.80$). An independent samples t-test revealed that the difference in mean fixation counts between the two groups was statistically significant ($t = 2.921$, $p = 0.009$).

**Table 5.** Average of fixation count of both groups

| | group | M | SD | Normality test | | t | p |
|---|---|---|---|---|---|---|---|
| | | | | Shapiro-Wilk | p | | |
| Synthesis | A | 116.10 | 32.865 | .970 | .785 | 2.921 | .009** |
| | A' | 79.80 | 21.540 | | | | |

Next, the number of gaze transitions between related AOIs in the visual models and numerical expressions was analyzed to investigate how organically students organized and coordinated their attention across the two representations (Table 6). The number of gaze transitions between related AOIs refers to the frequency with which students shifted their gaze between corresponding parts of the visual model and the numerical expression at each step of the algorithm.

The analysis revealed that, across all tasks, Group A generally exhibited a higher average number of gaze transitions compared to Group A'. In particular, for Addition Task 3, Group A showed an average of 6.0 transitions, whereas Group A' averaged 4.4 transitions—a statistically significant difference ($t = 2.097$, $p = 0.050$). Similarly, for Subtraction Task 2, Group A showed an average of 5.8 transitions, while Group A' averaged 3.7 transitions, also indicating a statistically significant difference ($t = 2.353$, $p = 0.030$).

When comparing the total number of transitions between related AOIs across all tasks, Group A averaged 31.8 transitions, whereas Group A' averaged 23.5 transitions. This suggests that students in Group A more frequently and actively shifted their gaze between corresponding AOIs—specifically M1↔N1, M2↔N2, and M3↔N3—compared to those in Group A' ($t = 2.560$, $p = 0.020$).

**Table 6.** Number of gaze movements between AOIs of both groups

| | Addition1 | Addition2 | Addition3 | Subtraction1 | Subtraction2 | Subtraction3 | Sum |
|---|---|---|---|---|---|---|---|
| A | 4.0 | 4.0 | 6.0 | 5.7 | 5.8 | 6.3 | 31.8 |
| A' | 3.2 | 2.9 | 4.4 | 4.3 | 3.7 | 5.0 | 23.5 |
| t | 1.00 | 1.941 | 2.097 | 1.521 | 2.353* | 1.433 | 2.560* |
| (p) | (.331) | (.068) | (.050) | (.146) | (.030) | (.169) | (.020) |

### Students' Understanding of Visual Models

The proportion of students in each group who were determined to have understood the visual models is presented in Table 7. For addition, 81.8% (9 students) in Group A were evaluated as having understood the visual model explaining the standard addition algorithm, compared to 70% (7 students) in Group A'. This indicates that Group A demonstrated a slightly higher level of understanding than Group A'.

In subtraction, the difference in understanding between the two groups was more pronounced. In Group A, 72.7% (8 students) were found to have understood the visual model explaining the standard subtraction algorithm, whereas only 50% (5 students) in Group A' met the same criterion.

**Table 7.** Percentage of students who understood the visual model

| | Percentage of students who understood addition | Percentage of students who understood Subtraction |
|---|---|---|
| Group A (11 students) | 81.8% (9 students) | 72.7% (8 students) |
| Group A' (10 students) | 70% (7 students) | 50% (5 students) |

In the interviews, students in Group A tended to explain the visual models by making explicit connections between the parts of the visual model and the numerical expression that were highlighted in the same color, as shown in Fig. 5. These students were able to relate the manipulation of unit blocks in the first step of the visual model to the ones place in the numerical expression. Moreover, after performing carrying or borrowing, they matched the quantity of base-ten blocks to the corresponding digits in the numerical expression, demonstrating an integrated understanding of the visual model and the algorithmic procedure.

**Fig. 5.** Interview scenes of Group A students

In contrast, students in Group A' tended to focus primarily on the numerical expressions, explaining the algorithmic steps without making explicit connections to the visual models, as shown in Fig. 6. For instance, Student 305 described the algorithmic procedure using only the digits in the final step of the numerical expression. Similarly, Student 311 broke down the standard algorithm into three steps but explained the operations solely in terms of numerical calculations, without reference to the visual model. These findings suggest that the presence of visual emphasis supports students in linking corresponding elements of the visual model and numerical expression, thereby facilitating integrated explanations.

**Fig. 6.** Interview scenes of Group A' students

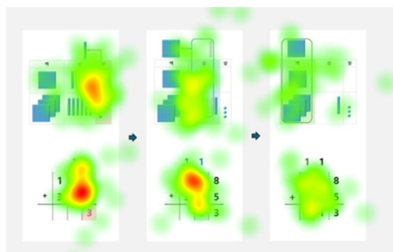
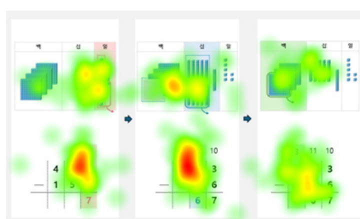
## Presentation Direction of Visual Models

### Eye-tracking Data Analysis

The heatmap comparison by task between Group B, which was presented with horizontally arranged visual models, and Group B', which received vertically arranged models, is shown in Table 8. Overall, Group B exhibited a relatively balanced distribution of gaze between the visual model (positioned above) and the numerical expression (below) at each step. In contrast, Group B' showed a tendency for gaze fixation to concentrate more heavily on the numerical expression (on the right) than on the visual model (on the left) at each step.

Although both groups generally exhibited more gaze fixation on the numerical expressions than on the visual models across all tasks, this pattern was more pronounced in Group B’ than in Group B. The results suggest that the vertical layout may lead to a stronger focus on the numerical expressions, potentially reducing students' attention to the visual models.

**Table 8.** Heatmap of group B and group B'

| | B (horizontal) | B‘ (vertical) |
|---|---|---|
| TAddition3 |  |  |
| Subtraction3 |  |  |

To quantitatively compare the differences in gaze distribution between the visual model and the numerical expression observed in the heatmaps, gaze duration ratios were calculated. The gaze duration ratio for the visual model was determined by computing the percentage of total task time spent fixating on the AOIs corresponding to the model (M1, M2, M3). Similarly, the gaze duration ratio for the numerical expression was calculated as the percentage of task time spent fixating on the corresponding AOIs (N1, N2, N3).

Table 9 presents the aggregated gaze duration ratios for the six tasks. The average gaze duration ratio for the visual model was higher in Group B (38.19%) than in Group B’ (27.07%), and this difference was statistically significant according to an independent samples t-test ($t = 2.569$, $p = 0.019$). In contrast, the average gaze duration ratio for the numerical expression was higher in Group B’ (55.99%) than in Group B (46.97%), but the difference was not statistically significant ($t = -1.603$, $p = 0.126$).

**Table 9.** Percentage of gaze durations for visual models and expressions in two groups

| | group | M | SD | Normality test | | t | *p* |
|---|---|---|---|---|---|---|---|
| | | | | Shapiro-Wilk | p | | |
| model | B | 38.19 | 10.61 | .953 | .412 | 2.569 | .019* |
| | B’ | 27.07 | 8.65 | | | | |
| expression | B | 46.97 | 11.80 | .958 | .509 | -1.603 | .126 |
| | B’ | 55.99 | 13.32 | | | | |

To identify statistically significant gaze transitions between AOIs, sequential analysis was conducted to generate a transition matrix. Sequential analysis, developed by Bakeman and Quera (1995), is an analytical method used to detect patterns of gaze transitions between defined AOIs. Based on the transition matrix, the probability of each transition was calculated to create a probability matrix, which reveals the likelihood of a gaze shifting from one AOI to another, thereby enabling a more detailed analysis of gaze movement tendencies.

Furthermore, the number of gaze transitions in the transition matrix was converted into z-scores. Transitions with a z-score greater than 1.96 ($p < .05$) were considered statistically significant. These significant transitions were visually represented with arrows in a gaze transition diagram to illustrate the directional flow of visual attention.

Figures 7 and 8 show the transition probability matrix and gaze transition diagram for Addition Task 3. In Group B, the following transitions were statistically significant: M1→N1 (0.72), N1→M1 (0.60), M2→N2 (0.56), N2→M2 (0.46), and N3→M3 (0.63). In contrast, Group B’ showed significant transitions in M1→N1 (0.50), M1→M2 (0.50), N1→N2 (0.53), and M3→N3 (0.45). Notably, the bidirectional transitions between corresponding steps of the visual model and numerical expression-such as M1↔N1 and M2↔N2-were more frequent and statistically significant in Group B than in Group B’, indicating a stronger integration of the model and equation in Group B.

| | M1 | M2 | M3 | N1 | N2 | N3 |
|---|---|---|---|---|---|---|
| M1 | | .24 | 0 | .72 | .04 | 0 |
| M2 | .26 | | .15 | 0 | .56 | .04 |
| M3 | .06 | .17 | | 0 | .06 | .72 |
| N1 | .60 | .15 | 0 | | .25 | 0 |
| N2 | .04 | .46 | .17 | .08 | | .25 |
| N3 | .06 | .13 | .63 | 0 | .19 | |

Group B

| | M1 | M2 | M3 | N1 | N2 | N3 |
|---|---|---|---|---|---|---|
| M1 | | .50 | 0 | .50 | 0 | 0 |
| M2 | .15 | | .31 | .08 | .46 | 0 |
| M3 | .09 | .27 | | .09 | .09 | .45 |
| N1 | .41 | .06 | 0 | | .53 | 0 |
| N2 | .06 | .17 | .17 | .17 | | .44 |
| N3 | .30 | .10 | .50 | 0 | .10 | |

Group B′

**Fig. 7.** Transition probability matrix for task addition 3

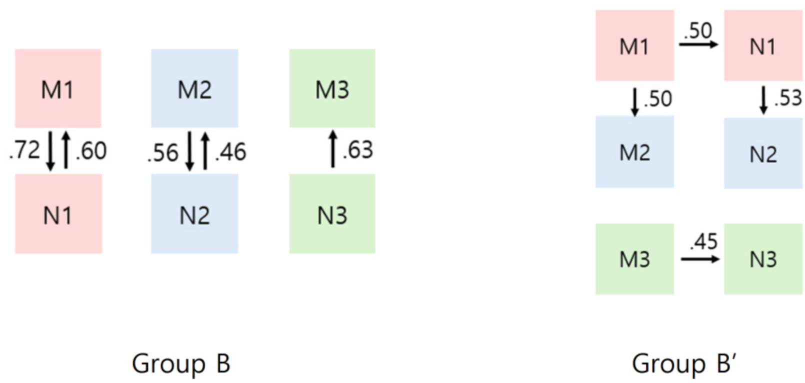



**Fig. 8.** Gaze transition diagram for task addition 3

**Students' Understanding of Visual Models**

In terms of understanding the addition algorithm, 90% (9 students) of Group B and 80% (8 students) of Group B' demonstrated a clear understanding of the visual model explaining the addition procedure. For subtraction, 80% (8 students) in Group B and 60% (6 students) in Group B' were found to have adequately understood the visual model representing the subtraction algorithm.

Given that Group B consistently showed slightly higher levels of understanding in both addition and subtraction compared to Group B', it can be inferred that presenting visual models in a horizontal layout may be more effective in supporting students' comprehension of standard arithmetic algorithms.

**Table 10.** Percentage of students who understood the visual model

| | Percentage of students who understood addition | Percentage of students who understood Subtraction |
|---|---|---|
| Group B (10 students) | 90% (9 students) | 80% (8 students) |
| Group B' (10 students) | 80% (8 students) | 60% (6 students) |

Students in Group B, who were presented with horizontally arranged visual models, tended to explain their reasoning by linking the visual model and the numerical expression—consistent with the frequent cross-referencing gaze patterns observed in the eye tracking data. For example, as shown in Fig. 9, Student 115 from Group B explained each step of the algorithm by focusing on the corresponding segment of the visual model, using their fingers to indicate quantities and explicitly connecting them to the digits in the numerical expression.

**Fig. 9.** Interview scenes of Group B student 115

In contrast, students in Group B', whose attention during the eye tracking assessment was more focused on the numerical expressions than on the visual models, also tended to explain their thinking by concentrating on the numerical expressions rather than the manipulation of base-ten blocks shown on the left. As illustrated in Fig. 10, Student 120's explanation of the addition process and Student 108's explanation of the subtraction process both focused solely on the right-hand side numerical expressions. When asked to explain their understanding, both students described the procedure by sequentially pointing to and interpreting the digits in the equations from top to bottom, without referencing the visual model.

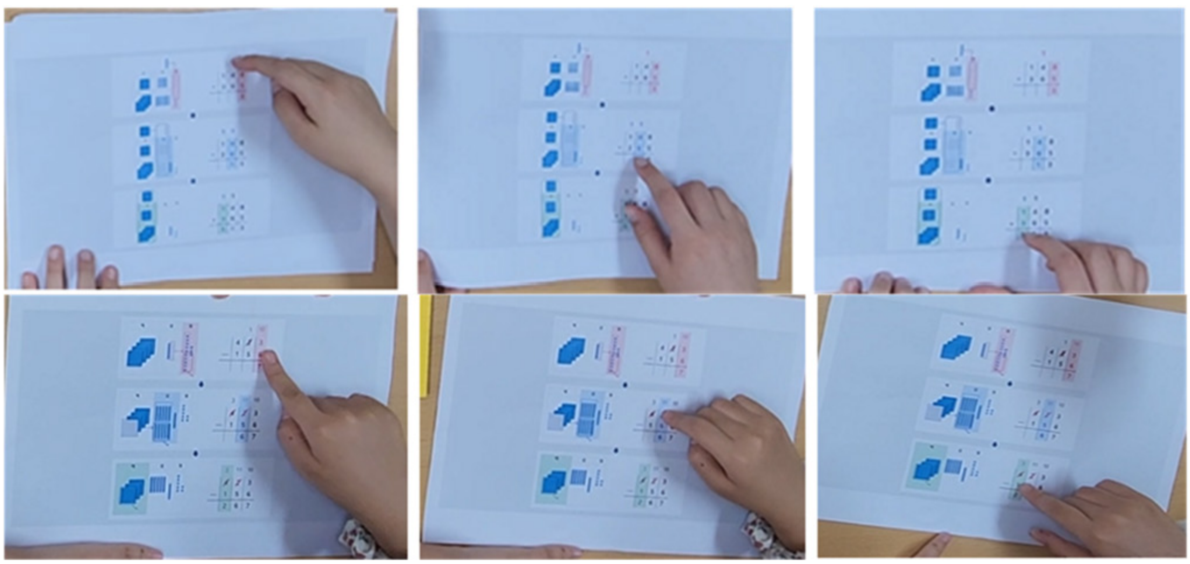


**Fig. 10.** Interview scenes of Group B' students

Synthesizing the findings from the eye tracking assessments and interview data, it was evident that the presentation direction of the visual models influenced students' ability to relate the models to the numerical expressions. Students in Group B, who were presented with visual models in a horizontal layout, tended to connect each step of the visual model with the corresponding part of the numerical expression in their explanations. This integration appeared to support a deeper understanding of the visual representations of the standard algorithm. In contrast, students in Group B', who were shown vertically arranged visual models, exhibited a tendency to focus more on the numerical expressions on the right, paying comparatively less attention to the visual models.

## Discussion

The experimental results of this study, when compared with prior research, yield the following implications:

First, visual emphasis within instructional visual models is effective in directing students' attention to critical areas and in facilitating their understanding by helping them connect the visual model with the corresponding numerical expressions. The comparison of heatmaps and fixation counts between the groups showed that students in Group A, who were presented with visually emphasized models, tended to focus their attention on the highlighted segments. In contrast, students in Group A', who were provided with non-emphasized models, exhibited more dispersed gaze patterns across the entire visual field and showed weaker fixation on the regions corresponding to each algorithmic step.

Moreover, students in Group A demonstrated more frequent gaze transitions between related regions of the visual model and the numerical expression, and this was reflected in their interviews, where they tended to explain the model and equation in an integrated manner. In contrast, some students in Group A' concentrated more on the numerical expressions and attempted to solve the base-ten block tasks through mental computation. During interviews, several of these students admitted to not paying close attention to the visual models or showed limited understanding of each step in the model.

These findings suggest that visual emphasis helps students focus their attention on relevant elements and supports their understanding of the meaning of each step by linking visual models with numerical representations. This aligns with Mayer's (2021) Signaling Principle in multimedia learning, which posits that adding cues to highlight the structure of essential content improves learning outcomes. It also supports the findings of Mautone and Mayer (2001), who showed that visual signaling (e.g., highlighting headings, topic sentences, and causal connectors) led to higher achievement in transfer tests, and of Jamet and Fernandez (2016), who demonstrated that aligning visual cues with verbal explanations enhances learning effectiveness.

Importantly, this study extends existing research by examining the effect of visual emphasis not in textual materials but in mathematical visual models and symbolic equations, specifically with elementary students. In contexts such as learning standard algorithms for addition and subtraction—where it is necessary to connect complex, multi-step visual models with symbolic notation—the use of visual emphasis has been shown to be beneficial. Accordingly, when designing visual models for textbooks, it is crucial to include visual emphasis on key steps, core concepts, and relevant regions to support students' conceptual understanding.

Second, presenting visual models in a horizontal layout was found to be effective in promoting students' balanced attention to both the visual model and the numerical expression, thereby supporting their integrated understanding. The heatmap analysis revealed that students in Group B exhibited a relatively even distribution of gaze between the visual model and the numerical expression at each step, while students in Group B' showed a strong tendency to focus more on the numerical expressions than on the model.

Additionally, the gaze duration ratio for the model region was higher in Group B than in Group B', whereas the gaze duration ratio for the numerical expression was higher in Group B'. This suggests that the horizontal layout helped students allocate their attention more evenly across both representations.

Sequential analysis of gaze flow and transition probabilities between AOIs further supported this interpretation. Group B demonstrated sequential gaze shifts between corresponding steps of the visual model and the numerical expression (e.g., M1 ↔ N1, M2 ↔ N2), indicating an integrated processing of the two representations. In contrast, Group B' students tended to first fixate on the numerical expression and then move frequently within that area, with less attention to the model.

This tendency was also reflected in the understanding assessments: Group B showed higher task success rates in both addition and subtraction. Interviews further confirmed that students in Group B tended to explain their reasoning by referring to both the model and the expression, whereas students in Group B' often relied on mental calculations and focused primarily on the right-hand side numerical expressions in a top-to-bottom manner.

Taken together, these findings suggest that a horizontal layout encourages students to attend to both visual and symbolic information and to make meaningful connections between them, which in turn supports their understanding of each step in the visual model. These findings are consistent with those of Kim and Lee (2017), who reported that students were better able to coordinate models and symbols when visual models and equations were presented vertically, and with Shin and Shin (2014), who found that image placement in science textbooks affected gaze patterns and fixation time.

This study adds to the literature by comparing horizontal and vertical arrangements of step-by-step visual models within mathematical instruction. It also aligns with Mayer's (2021) Spatial Contiguity Principle, which asserts that placing related information close together facilitates cognitive processing. In the horizontal layout used in this study, the proximity between corresponding regions of the visual model and the numerical expression may have supported students' integration of the two sources of information.

Therefore, when designing textbook materials, the spatial arrangement of visual models should be carefully considered. Aligning the layout with students' natural gaze flow and supporting connections between models and equations can enhance comprehension, particularly in topics involving multi-step mathematical procedures.

## Implications

Based on the findings and discussion of this study, the following implications are suggested:

First, when presenting visual models in textbooks, it is essential to include visual emphasis on key steps, core concepts, and relevant regions. Visual emphasis plays a crucial role in directing students' attention to each stage of the addition and subtraction algorithms and helps them naturally understand the connections between the visual model and the numerical expression. Therefore, mathematics textbooks should be designed to highlight important steps and concepts during computational procedures, enabling students to relate the visual model to the symbolic representation and thereby gain a clearer understanding of the underlying principles of standard algorithms.

Second, the direction in which visual models are presented should be carefully considered with respect to students' attention and the connection to numerical expressions. A horizontal layout of visual models allows students to view the model and the equation in a more balanced and integrated manner, thereby supporting their comprehension of each step in the standard algorithm. Accordingly, textbook design should adopt a horizontal presentation of visual models to enhance learning effectiveness, particularly in instructional contexts involving multi-step operations such as addition and subtraction.

This study analyzed students' gaze on the visual model for the addition and subtraction algorithm of natural numbers using eye tracking technology. However, since this study focused on a specific mathematical topic (addition and subtraction of whole numbers), further research is needed to generalize the results, such as exploring similar effects in other mathematical topics (e.g., multiplication, fractions) or in digital learning environments.

## Conflicts of Interest

The author(s) declared no conflicts of interest.

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