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The Analysis of 3rd Graders' Multiplicative Thinking

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초등학교 3학년 학생들의 곱셈적 사고 분석

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<ABSTRACT>

The purpose of this study is to analyze 3rd graders' thinking about multiplication by using eye tracking techniques. The researchers presented 1-digit multiplication questions along with a visual representation and then asked the participants whether the multiplication and the visual representations were equivalent. For the pre-test, we tracked 13 students and selected 7 students who had additive thinking about multiplication. 4-weeks of instruction was carried out to ensure they understand the meaning of multiplication. After the instruction we tracked their eye movements with the similar 1-digit multiplication tests. The students' eye tracking data showed that their thinking about multiplication changed from additive thinking to multiplicative thinking. Through the eye tracking technique, we can clearly identify changes of students' thinking about multiplication.

Keywords : Multiplication, eye tracking, multiplicative thinking, additive thinking, visual representations

I. Introduction

The operation rules are basic mathematics concept and understanding the principles of computation and becoming proficient in operation rules are the main educational purposes of elementary school mathematics. Among them, multiplication is the most important foundation to learn many subsequent mathematics contents because of its proportionality(Chong, 2013).

Students need to know how to solve problems through multiplicative thinking to clearly understand the meaning of multiplication and to use it competently in different situations. Some researchers insist in studying the identification of multiplicative thinking that is developed from additive thinking in grades 1–5. Results have shown that the multiplicative thinking can clearly be distinguished

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from additive thinking and is found to develop slowly, therefore it is important to help children to transfer from additive thinking to multiplication (Clark & Kamii, 1996).

Despite the importance of multiplicative thinking, many students are still trying to solve multiplication problems by using the concept of repeated addition. It was found that 94.8 percent of the fourth to sixth-grade students who were observed answered multiplication problems with additional thinking in their multiplication test(Han & Ryu, 2008). According to research about multiplicative thinking of first to fifth grade children, many children still have additive thinking even though they learn multiplication in their school curriculum. So, it is necessary to develop multiplicative thinking through varied activity including manipulative activities(Lee, 2001). It was found in study on the development of understanding of multiplication that very few students used multiplicative thinking to solve problems, so it was necessary to develop multiplicative thinking beyond a commonly known strategy(Anghileri, 1989).

Nowadays the study of multiplicative thinking and instruction of multiplication has been conducted in a variety of ways(e.g. Sherin & Fuson, 2005; Dejarnette, Walczak & González, 2014; Hurst, 2015; Kouba, 1989). However, in these studies, the measurement of a student's multiplicative thinking was indirectly measured by the correct answer rate or the answers of the students. Some researchers were able to confirm students' multiplicative strategies more directly by showing that the strategy of solving multiplication can be measured with eye tracking technique(Bolden, Barmby, Raine & Gardner, 2015).

However, the prior study focused only on classifying multiplication strategies for students, so it has not yet been revealed whether students are good at solving problems according to their strategies. Thus, in our studies, when students solve multiplication problems, we will analyze our eye tracking technique along with their answers to identify what they are thinking, and we will provide a four-weeks of multiplication instruction to show how students' thinking has changed.

II. Method

1. Participants

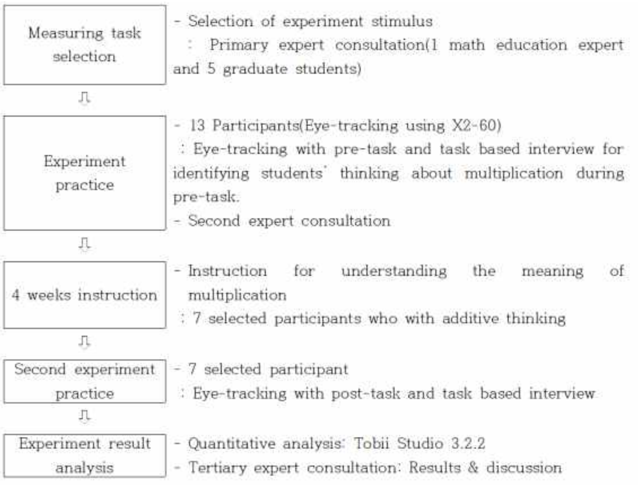
15 third grade students (9 females, 6 males) from a small elementary school in Y City participated in this study. This elementary school had underachieving students as determined through a national academic achievement test, so we could compare underachieving students with high achieving students at the same time.

7 underachieving students and six high achieving students were recommended by their homeroom teachers based on their academic achievement test. The recommended students were not all wearing glasses so that their eye movements could be traced. Since multiplication in Korean education is taught in the second-grade curriculum, students have already learned the meaning of multiplication and have learned the multiplication table.

2. Procedure

For this study, an expert group was formed with one math education expert and five graduate students of elementary mathematics education. Expert consultation was held to increase the validity and reliability of task selection, experimental design, and analysis of test results.

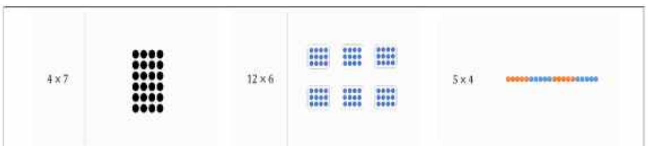
Pre-tasks were selected through primary expert consultation, and the main experiment was carried out using pre-tasks. After the main experiment, the task-based interview was conducted to identify how the students solved their tasks. Based on the results of the main experiment, 7 students who have additive thinking were selected and were given 4-weeks of instruction about the meaning of multiplication. After the 7 students were tested using evaluation technique, the results and implications of this study were derived by analyzing the data. A summary of the research procedures is given [Figure 1].



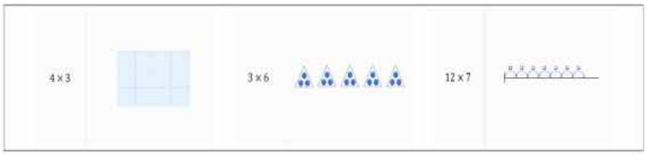
[Figure 1] Diverging stacked bar charts for student experience using campus and external e-learning systems

3. Task

We have developed 16 tasks and presented 1-digit multiplication along with a visual representation. Then we asked the test participants whether the multiplication and visual representations are equivalent. Types of visual representations were generally used in an array representation, group representations and line representations. As [Figure 2] shows, each visual representation was presented to participants with variations as [Figure 3](Chong, 2013).



[Figure 2] Types of visual representations of multiplication; array representations (at left), groups representations (at middle), line representations (at right)



[Figure 3] Variations of visual representations of multiplication; array representations (at left), groups representations (at middle), line representations (at right)

Pre-tasks and a post-task were composed of the same type of problems. However, considering that students are in third grade, the post tasks presented one question per each visual representation that was beyond the scope of the multiplication table.

4. Data Collection

In this study, Tobii's X2-60 device was used to track the eyes movement and the tasks were presented on a screen with a 24-inch monitor (1920 pixel * 1080 pixel). The distance between the monitors and participants was maintained at about 70cm. During the experiment, the movements of participants were minimized to ensure more accurate measurements.

Participants who did not have proper calibration and validation and participants with less than 60 % tracing ratio were excluded. However, one student was included in the results analysis through expert consultation. Since this is a qualitative study, this student's results were in even though the student's tracking ratio was low because it best shows the visualization of the change after the 4-weeks of instruction.

5. Data Analysis

After the participant solved the task, a task-based interview was conducted to see what strategy they were using. It was found that students used additive thinking in which they interpret multiplication through repeated addition, without understanding the structure of multiplication(Clark &

Kamii, 1996). Thus, participants who solved all visual representations through counting or adding were classified as part of the additive thinking group. Participants who solved tasks by understanding the structure of multiplication were classified as the multiplicative thinking group. The results were derived by analysing which aspects of the eye tracking data they had in common.

The Tobii Studio 3.3.2 program was used for eye tracking techniques. Among many visual models, eye tracking techniques were used to focus on the most visible array models. For an array model, the eye tracking analysis included a 'count all strategy' and a 'count axes strategy'(Bojko, 2013).

For the results, we held a second expert consultation to classify students' thinking of multiplication as additive or multiplicative through cross-checking task-based interview and eye tracking data.

III. Results

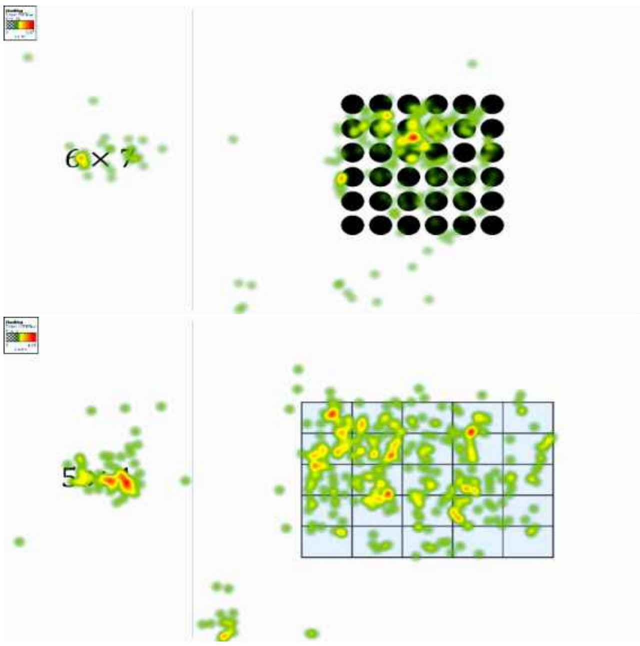
1. Analysis by Heatmap between Students' Thinking about Multiplication

After conducting the main experiment with 15 participants, the analysis was conducted on 13 students, excluding those with less than 60% of the tracking ratio. 7 of the 13 students had additive thinking, and six knew the meaning of multiplication and solved the task with multiplicative thinking.

After the main experiment, we did a task-based interview with each of the students to see how they solved the task. Participants P0102, P120 had difficulty with solving 1-digit multiplication because they were unable to remember multiplication tables and used a count all strategy in all the visual representations. Participant P0201 interviewed with no explanation for how to interpret the visual representations, so it was considered a count all strategy for all the visual representations. Participants P110, P0214 explained that the multiplication table was used to calculate the multiplication and count the same numbers in a visual representation. Participants P0204, P0222 understood the meaning of multiplication as only repeated addition, and just answered the task using the multiplication table. Participant P0219 knew the meaning of multiplication as a few batches but was classified as having additive thinking because he uses count all strategy in interpreting the visual representations.

We checked the recordings of the participants categorized in additive thinking group with Heatmaps [11], which show the distant distribution of where the participants looked at on the screen during a given time window. See [Figure 4].

Students with additive thinking chose a count all strategy in interpreting visual models, so it is important to know that eye fixation points existed on every part of the representations on Heatmap.

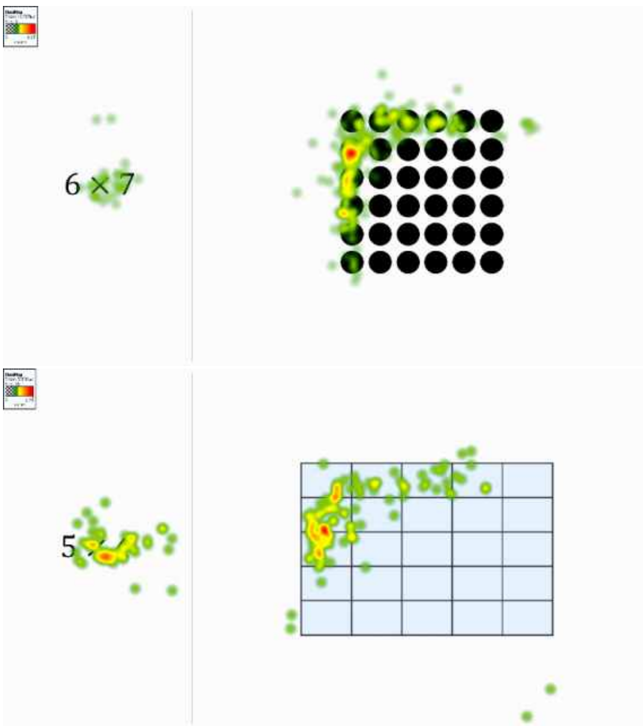


[Figure 4] Heatmap of the participants categorized in additive thinking on array task.

We also did a task-based interview. participants P0216, P0217, who understood the meaning of multiplication well and solved the task without any calculations, were classified as having multiplicative thinking. Since participants P0101, P0116, P0122 and P0102 calculated the answer to solve the task, but understood the meaning of multiplication as a few batches, so they were classified as having multiplicative thinking.

We also checked the recordings of the participants categorized in multiplicative thinking with Heatmap. See [Figure. 5]

Participants who have multiplicative thinking understood the meaning of multiplication as a few batches. They used a count-axes strategy, not the count-all strategy to interpret the visual representation; therefore, it is important to know that eye fixation points existed on axes of the representations on Heatmap.



[Figure 5] Heatmap of the participants categorized in multiplitative thinking on array task.

We compared the students' answers to thinking about multiplication to see if there was any connection between them. See <Table 1>. For students who have multiplicative thinking, they were able to answer more than 11 of the 16 questions correctly. On the contrary, most students who have additive thinking had a correct answer rate of less than 60 percent of the 16 questions answered. However, students who have additive thinking such as P0214, P0219, and P0222 are also found to have high rates of correct answers. These students mean they are accustomed to solving the problems of multiplication back to addition(Clark & Kamii, 1996). In other words, by observing only the correct answer rate for the written evaluation, it is impossible to classify the students' thinking about multiplication.

<Table 1> The thinking about multiplication and the answers of the participants

Participant	Thinking about multiplication	Answers
P0101	Multiplicative	12/16
P0102	Additive	8/16
P0110	Additive	10/16
P0116	Multiplicative	16/16
P0120	Multiplicative	16/16
P0122	Multiplicative	11/16
P0204	Additive	9/16
P0210	Additive	0/16
P0214	Additive	13/16
P0216	Multiplicative	13/16
P0217	Multiplicative	15/16
P0219	Additive	14/16
P0222	Additive	13/16

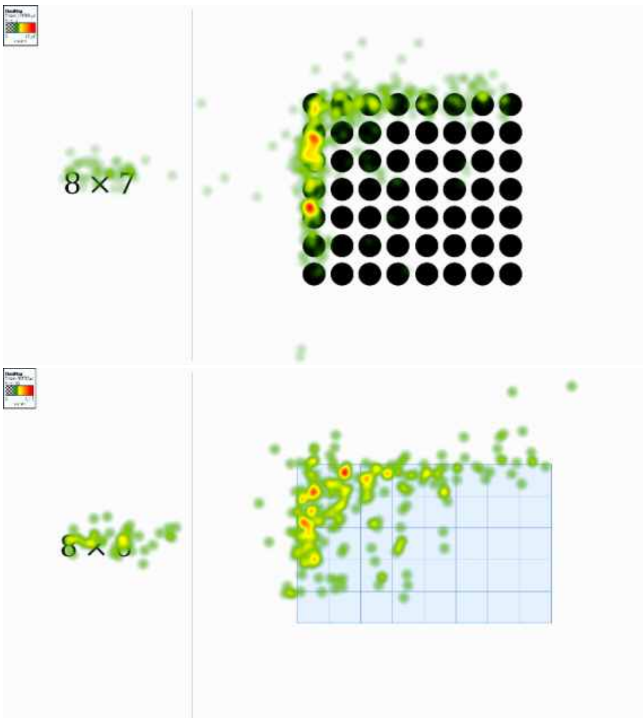
2. Comparison of Students' Thinking about Multiplication after 4-week Instruction

It is said that when thinking about multiplication, it is important to know that multiplication utilizes the concept of a few batches, not by repeated addition(Kang, 2009).

7 participants classified as having additive thinking were given four weeks of instruction highlighting the meaning of multiplication. We

observed how the thoughts of multiplication changed. After the 4-weeks of instruction, participants P0219, P0214, P0201 and P0204 understood the meaning of multiplication, solved and explained tasks well. Participants P0222, P0102 and P110 answered in the task-based interview with a good understanding of multiplication. However, after analyzing the Gaze-plot, it was found that in most cases they use the count-axes strategy, but in a few task they still use the count-all strategy, thus they can be seen as transitional stages of additive thinking and multiplicative thinking.

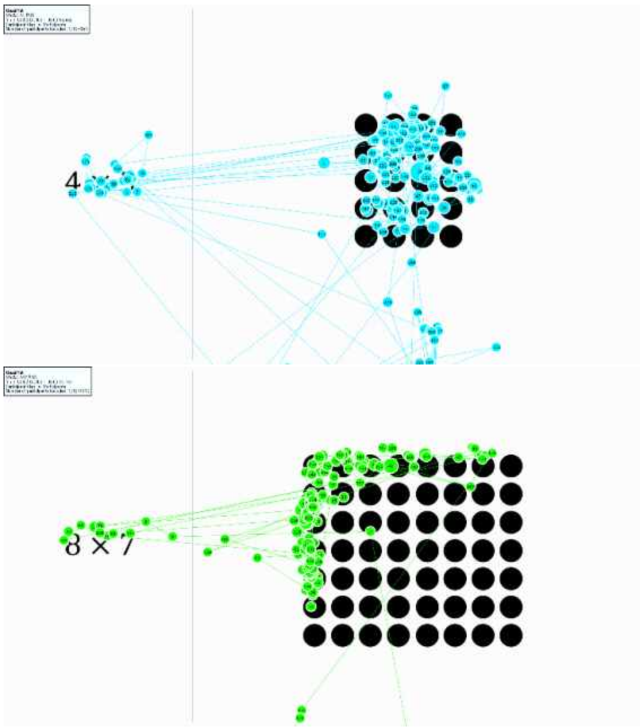
To help visualize these findings, we considered the recordings of 7 selected participants with Heatmap. See [Figure 6]



[Figure 6] Heatmap of 7 selected participants on array task after 4-week instruction

Participant P0201 was the most dramatic example of a change. In the pre-task, she could not answer the task at all and did not know how to interpret the visual representations, so she counted it one by one and solved it. However, after the four weeks of instruction she used the terms of a few batches to explain how she solved the post task. To see this visually, we conducted a Gaze-plot analysis of participant P0201. See [Figure 7].

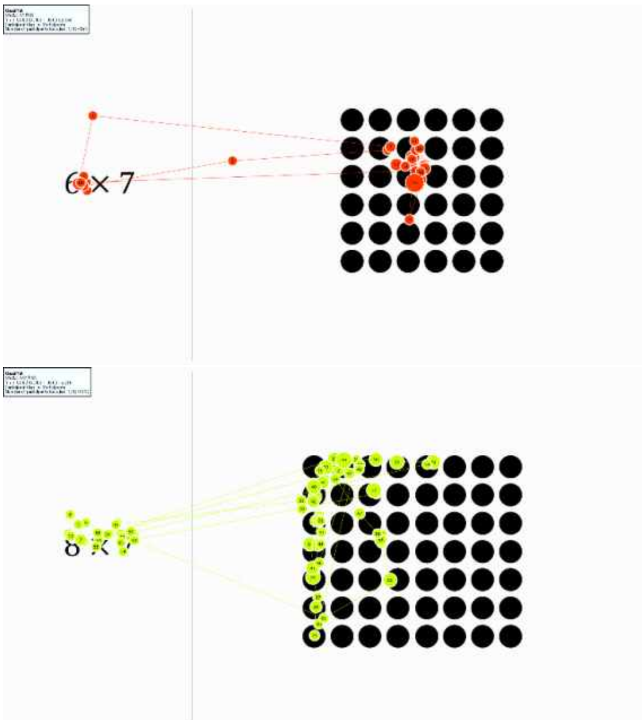
In the pre-task, she used a count all strategy, but in the post task, she turned it into a count axes strategy.



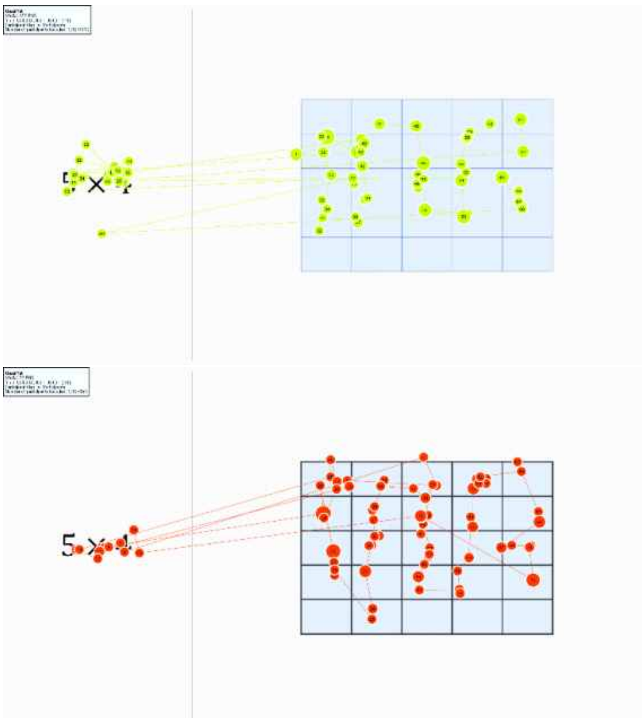
[Figure 7] Comparison of gaze-plot of P0210 between pre-task (top) and post-task (bottom)

Participant P0110 is a representative example of transitional stage, changing into multiplicative thinking. As with other participants, in the pre-task he used a count-all strategy to interpret the visual representations, but in the post-task it changed to a count-axes strategy. However, it turns out that he still uses a count all strategy in a few tasks. To see this visually, we conducted a gaze-plot analysis of participant P0110.

Participant P0110 used a count axes strategy in the points array representations, See [Figure 8]. However, he used a count all strategy in the rectangle array representations. See [Figure 9].



[Figure 8] Comparison of gaze-plot of P0110 between pre-task (top) and post-task (bottom)



[Figure 9] Comparison of gaze-plot of P0110 between pre-task (top) and post-task (bottom)

IV. Discussion

The use of eye tracking techniques made it more visible and made it more clearly classify the students' thinking about multiplication. It is known that underachieving students have trouble explaining the process in which they solved the problem(Bailey, 2002). However, as we saw earlier in <Table 1>, it is difficult to classify students' thinking about multiplication only with a percentage of correct answers. Thus, analyzing eye tracking data together will help classify students' thinking about multiplication. Several researchers have already identified the strategy of interpreting the visual representations of multiplication with eye tracking techniques(Bolden et al., 2015). In this study, we identified the strategies of interpreting the visual representations of multiplication and matched which the students' thinking about multiplication by comparing eye tracking data and data from the task-based interview. As a result, the count-all strategy was demonstrated by participants who have additive thinking, and the count axes strategy was shown at participant who have multiplicative thinking. This may be a natural result when we consider multiplicative thinking as changing sets of singletons to composite units(Steffe, 1994). Although we need to observe the gaze-plot of the students carefully, eye tracking techniques would allow researchers to better visualize the student's thinking. So, this study showed that the eye tracking techniques have benefits of providing clearer evidence of the students' thinking about multiplication.

Given that this is a qualitative study to explore the possibilities of using eye tracking technique in education, we propose a further quantitative follow-up study to support this. In addition, to classify the thinking of multiplication for students using eye tracking techniques, it is important to set a threshold that distinguishes between additive thinking and multiplicative thinking in the gaze-plot pattern.

Through the instruction highlighting the meaning of multiplication, students can convert their thinking of multiplication into multiplicative thinking. Korean third-graders are already taught to learn multiplication in the second grade and to memorize multiplication tables. Nevertheless, some of the underachieving students do not understand the meaning of multiplication, causing a huge disadvantage to their understanding of more complex mathematics later in their education. Therefore, when guiding multiplication, it is important to cultivate multiplicative thinking through instruction that emphasizes the meaning of multiplication. Although it was just four weeks of instruction, the results in this study showed a change in thinking about multiplication. Therefore, if students are systematically taught instruction that emphasizes the meaning of multiplication, they can develop multiplicative thinking.

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